

Finite-temperature behaviour of Dirac eigenspectrum in lattice QCD

Towards the UV dimension of eigenmodes in the IR phase

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One question for this talk

Dirac eigenmodes revealed an exotic infrared structure in thermal
QCD

Do they also carry a non-trivial ultraviolet dimensional structure?

Main idea

We study the scaling of effective mode volume as the lattice cutoff is removed, aiming at the first determination of d_{UV} in this setting.

Why Dirac eigenmodes?

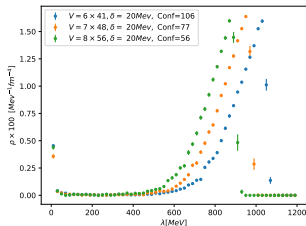
- Dirac spectra provide non-perturbative probes of QCD.
- Linked to chiral symmetry breaking, topology, and local vacuum structure.
- At finite temperature, low Dirac modes reveal a new organizing principle: **IR–bulk separation**.

Physical intuition

Instead of only asking how many eigenmodes exist at a scale, we ask how much space the corresponding eigenmodes effectively occupy.

Thermal QCD: the IR phase

- Above the thermal transition, the low Dirac spectrum develops a characteristic structure:
 - IR peak,
 - depleted plateau,
 - UV rise.
- The IR component shows unusual localization and effective low-dimensional behavior.
- This suggests an Anderson-like structure with two critical points.

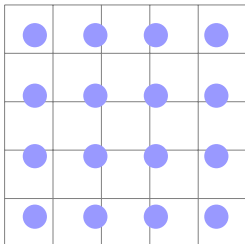


Spectral density over broad range

$$\rho(\lambda) = \frac{\langle n(\lambda, d\lambda) \rangle}{V_4 d\lambda} \quad \longrightarrow \quad \rho(\lambda, \delta) = \frac{\sigma(\lambda + \delta/2) - \sigma(\lambda - \delta/2)}{V\delta}.$$

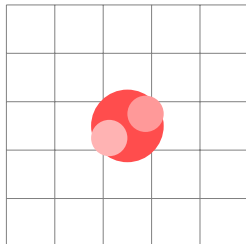
Extended vs Localized Modes (Geometry)

Extended mode



Spreads over whole system

Localized mode

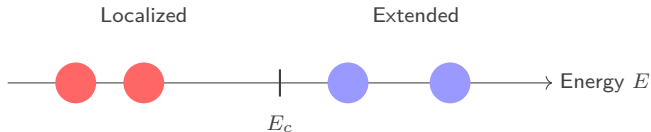


Confined to small region

Key idea

Dirac eigenmodes in QCD can behave like either of these

Anderson transition in Disordered Systems



Critical energy E_c separates:

- localized states
- extended states

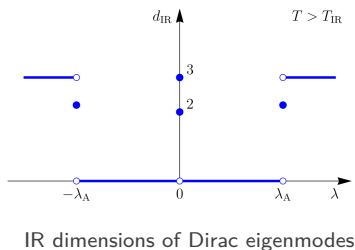
At E_c : critical / fractal behaviour

Anderson transition

Change from extended \leftrightarrow localized states

Anderson-like Behaviour in QCD

- Dirac eigenmodes behave like wavefunctions
- Gauge fields act as a kind of "disorder"
- Different regions of the spectrum:
 - point-like/localized: $d_{\text{IR}} \simeq 0$,
 - surface-like: $d_{\text{IR}} \simeq 2$,
 - extended in space: $d_{\text{IR}} \simeq 3$,



Key idea

QCD vacuum acts as a disordered medium where eigenmodes can localize. The Dirac spectrum shows transitions similar to Anderson localization.

IR versus UV dimension

IR scaling

$$V_{\text{eff}}(L, a) \propto L^{d_{\text{IR}}(a)} \quad L \rightarrow \infty$$

- probes long-distance support,
- already studied in the IR phase,
- reveals low-dimensional IR structure.

UV scaling

$$V_{\text{eff}}(L, a) \propto a^{d_{\text{UV}}(L)} \quad a \rightarrow 0$$

- probes cutoff response,
- not yet determined in QCD,
- central goal of this project.

Project goal

Determine the UV dimension d_{UV} of low-lying overlap Dirac eigenmodes in the IR phase of pure-gauge QCD.

- Temperature: $T = 1.12 T_c$.
- Theory: $N_f = 0$ QCD / pure Yang-Mills.
- Observable: effective volume and effective length of low-lying overlap modes.
- Physics target: complete the dimensional picture of the IR phase.

Label	N_τ	N_s	a [fm]	β	configurations
A	6	41	0.0992	5.964	106 present
B	7	48	0.0840	6.054	77 present
C	8	56	0.0744	6.138	56 present

Strategy

Use several lattice spacings at fixed temperature to extract cutoff scaling.

Overlap operator

$$D_{\text{ov}} = \frac{1}{2a} (1 + \gamma_5 \text{sign}(H)), \quad H = \gamma_5 D_W.$$

- Clean chiral properties make overlap modes especially suitable for spectral and topological questions.
- Main numerical difficulty: applying the sign function $\text{sign}(H)$.
- Workflow:
 - rational approximation to $\text{sign}(H)$,
 - Arnoldi algorithm for low modes of D_{ov} .

From eigenmodes to effective volume

For a normalized eigenmode $\psi(x_i)$ define the probability vector

$$P = (p_1, \dots, p_N), \quad p_i = \psi^\dagger \psi(x_i).$$

Map it to a counting vector

$$C = NP, \quad \sum_i c_i = N.$$

The effective number is

$$N^*[C] = \sum_{i=1}^N \min(c_i, 1),$$

so that

$$V_{\text{eff}} = N^* a^4.$$

Extracting d_{UV}

At fixed physical temperature and eigenvalue scale,

$$\langle N^*(a) \rangle_{\lambda, L, T} \propto a^{-d_{UV}(\lambda, L, T)}.$$

Equivalently,

$$V_{\text{eff}}(a) \propto a^{4-d_{UV}(\lambda, L, T)}.$$

Interpretation

- $d_{UV} = 4$: support scales as a four-dimensional object under cutoff removal.
- $d_{UV} < 4$: non-trivial UV-dimensional structure.

A second diagnostic: effective length

Define a mode radius by the probability-weighted distance from the maximum:

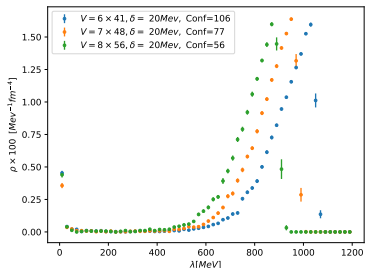
$$\ell[\psi] = \sum_i p_i |x_m - x_i|, \quad p_m \geq p_i \forall i.$$

The UV index is monitored through

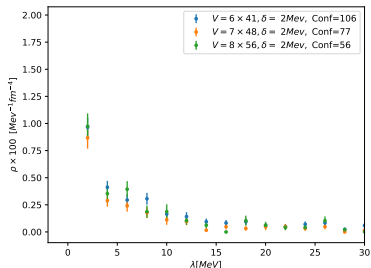
$$\langle \mathcal{L}[\psi] \rangle_{a,L,\lambda} \propto a^{1-\gamma(L,\lambda)}, \quad a \rightarrow 0.$$

- γ complements d_{UV} .
- It helps detect unusual cutoff dependence of the mode radius.

Preliminary result I: spectral density



Spectral density over broad range

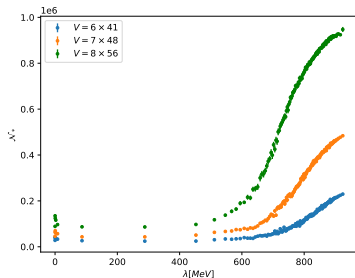


Zoom into IR region.

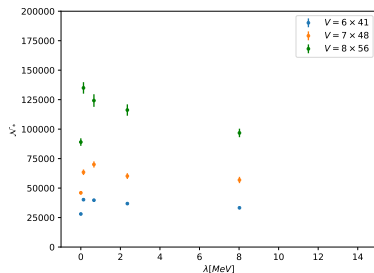
Observation

IR peak, depleted plateau, and UV rise are consistent with the IR phase picture.

Preliminary result II: effective volume



Effective volume N^* versus eigenvalue scale



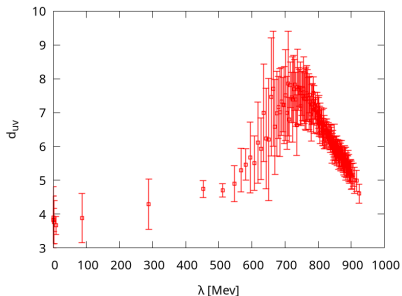
Zoom of N^* in the IR region

Use in analysis

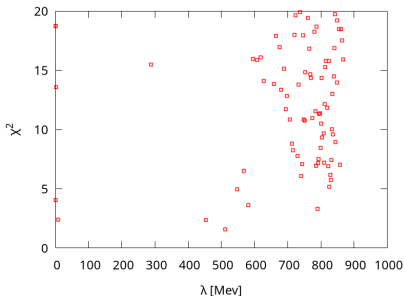
Binned effective volumes are the input to the UV scaling fits.

$$\langle N^*(a) \rangle_{\lambda, L, T} \propto a^{-d_{UV}(\lambda, L, T)}, \quad V_{\text{eff}}(L, a) \propto a^{4-d_{UV}(L)} \quad a \rightarrow 0$$

Preliminary result III: UV dimension



Extracted d_{uv} versus eigenvalue scale

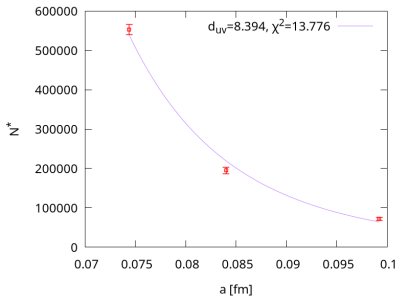


Fit quality, $\chi^2/d.o.f$ versus eigenvalue scale

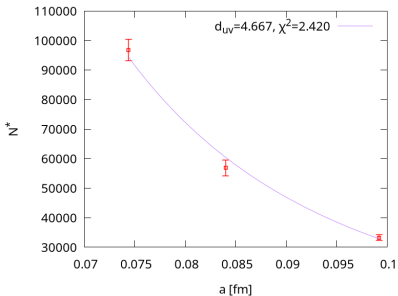
Current indication

Preliminary fits suggest $d_{UV} \approx 4$ in the IR and plateau regions, but both systematic and statistical errors are still significant.

Where the systematics show up



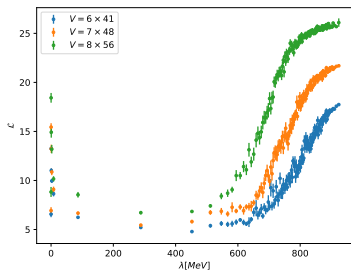
Example UV-bin fit, e.g. $\lambda \simeq 732$ MeV



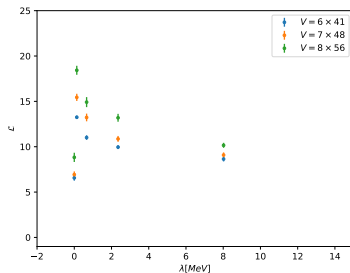
Example IR-bin fit, e.g. $\lambda \simeq 8$ MeV.

- Three lattice spacings are not enough for a controlled continuum statement.

Preliminary result IV: effective length



Effective length \mathcal{L} versus eigenvalue scale



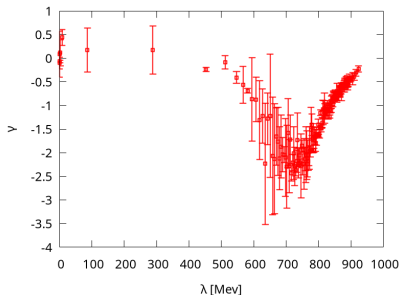
UV scaling index γ and fit quality

Use in analysis

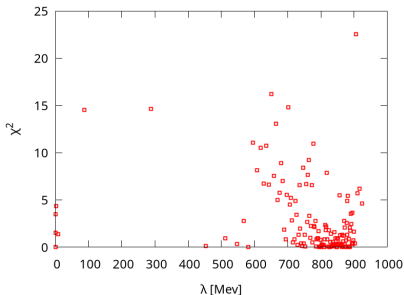
Binned effective volumes are the input to the UV scaling fits.

$$\langle \mathcal{L}[\psi] \rangle_{a,L,\lambda} \propto a^{1-\gamma(L,\lambda)}, \quad a \rightarrow 0.$$

Preliminary result IV: Length scaling exponent γ



Example UV-bin fit, e.g. $\lambda \simeq 732$ MeV

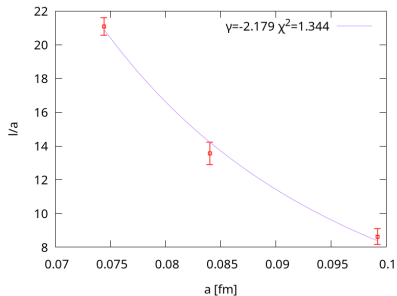


Example IR-bin fit, e.g. $\lambda \simeq 8$ MeV.

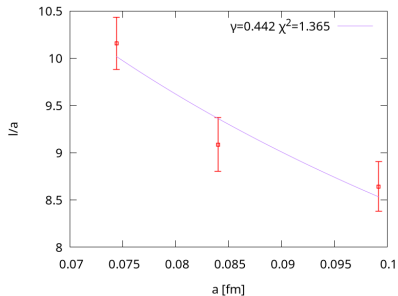
Current indication

The length analysis is broadly consistent with ordinary UV scaling, but again requires better control of systematics.

Preliminary result IV: Fits for the exponent γ



Example UV-bin fit, e.g. $\lambda \simeq 732$ MeV



Example IR-bin fit, e.g. $\lambda \simeq 8$ MeV.

- Three lattice spacings are not enough for a controlled continuum statement.

Possible Origin of Fit Instabilities

- Near transition regions:
 - no single scaling behaviour
 - effective dimension ill-defined

Interpretation

Instabilities may reflect Anderson-like critical behaviour

If $d_{UV} \simeq 4$, the IR phase appears exotic in the infrared, but standard in its short-distance scaling.

- Non-trivial IR dimensions remain a signature of the IR component.
- UV scaling tests whether this exotic structure persists into the cutoff regime.
- The current evidence points to ordinary UV behavior, but more statistics and a finer lattice spacing are needed.

Takeaway

- Thermal QCD contains a non-trivial IR spectral structure.
- Effective dimension gives a quantitative way to describe the support of Dirac modes.
- This project targets the first systematic computation of d_{UV} for overlap modes in the IR phase.
- Preliminary picture:
 - $d_{UV} \approx 4$, $\gamma \approx 0$
 - ordinary UV scaling despite exotic IR behavior,
 - systematics must be controlled with ensemble D and increased statistics.

Final message

The UV dimension is the missing piece in the dimensional characterization of the IR phase.

Thank you

Questions?