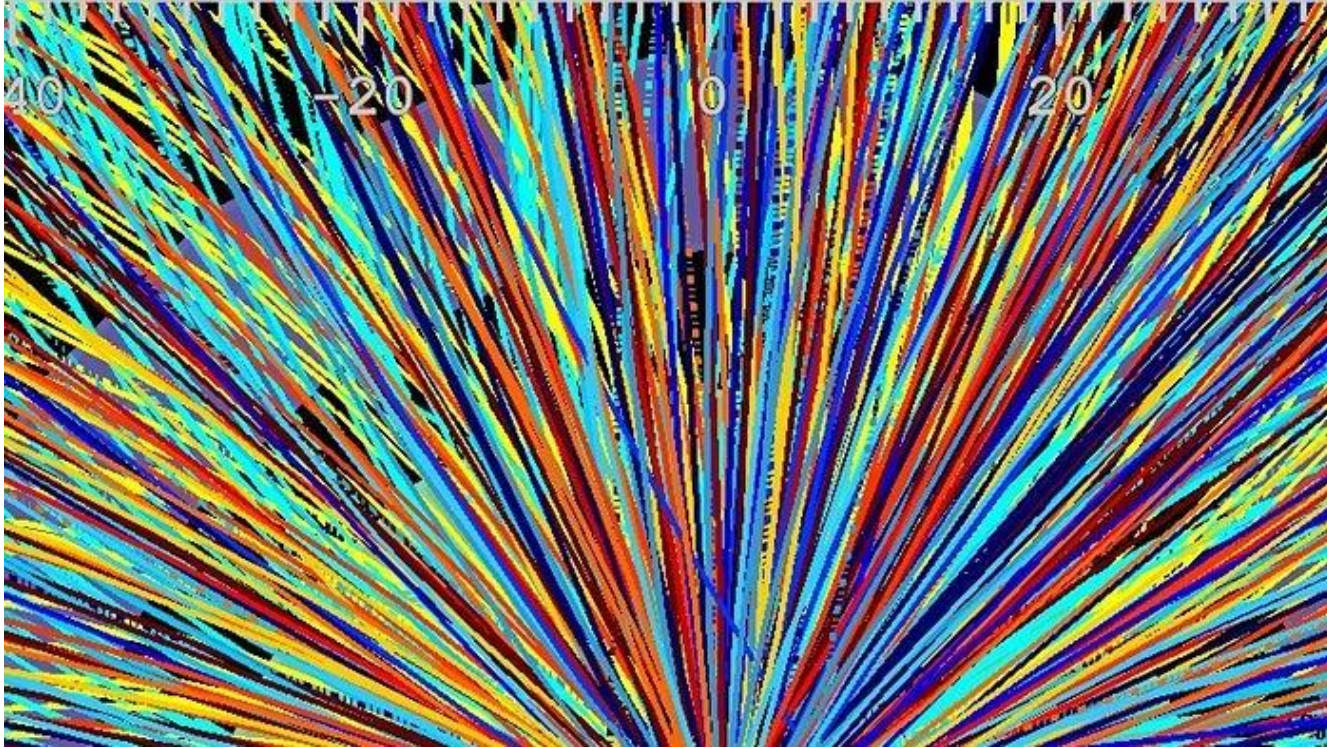
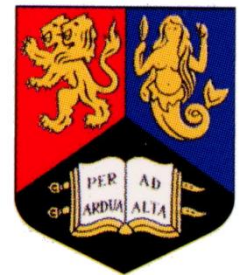




HI Physics Overview

R. Lietava

The University of Birmingham



Bratislava before the Iron Curtain Fall

On the Time Dependence of J/ψ Suppression by the Quark - Gluon Plasma

I. Horvath (Comenius U.), P. Lichard (Comenius U.), R. Lietava (Bratislava, Inst. Phys.), A. Nogova (Bratislava, Inst. Phys.), Jan Pisut (CERN)

Jul, 1988

4 pages

Published in: *Phys.Lett.B* 214 (1988) 237

Published: 1988

DOI: [10.1016/0370-2693\(88\)91475-X](https://doi.org/10.1016/0370-2693(88)91475-X)

Report number: CERN-TH-5125/88

View in: [ADS Abstract Service](#), [CERN Document Server](#)

 cite

 claim

 reference search

 24 citations

Content

- LHC intro
- Heavy Ion (HI) experiments
- Standard Model (SM) of HI
- Observables
- Connection to Lattice QCD
- Collectivity in small systems

HI experiments

- RHIC (Brookhaven, USA)
 - STAR and sPHENIX :
 - Physics like LHC but lower energy
 - Energy scan => QCD phase diagram critical point
- FAIR (Darmstadt, Germany)
 - Cold Baryon Matter (CBM)
- NICA (JINR, Russia) – baryon matter
- SPS (CERN, Switzerland)
 - Fixed target
 - Energy scan

HI at LHC

- ALICE – dedicated HI experiment
 - Optimised for low transverse momentum p_T
 - and PID
 - Soft probes at QCD scale
 - Heavy flavours down to zero p_T
- ATLAS, CMS – general experiments for Higgs and new physics
 - Optimised for high $p_T > 10$ GeV
 - Jet quenching, photon/Z/W, heavy flavours
- LHCb – heavy flavour central-forward physics

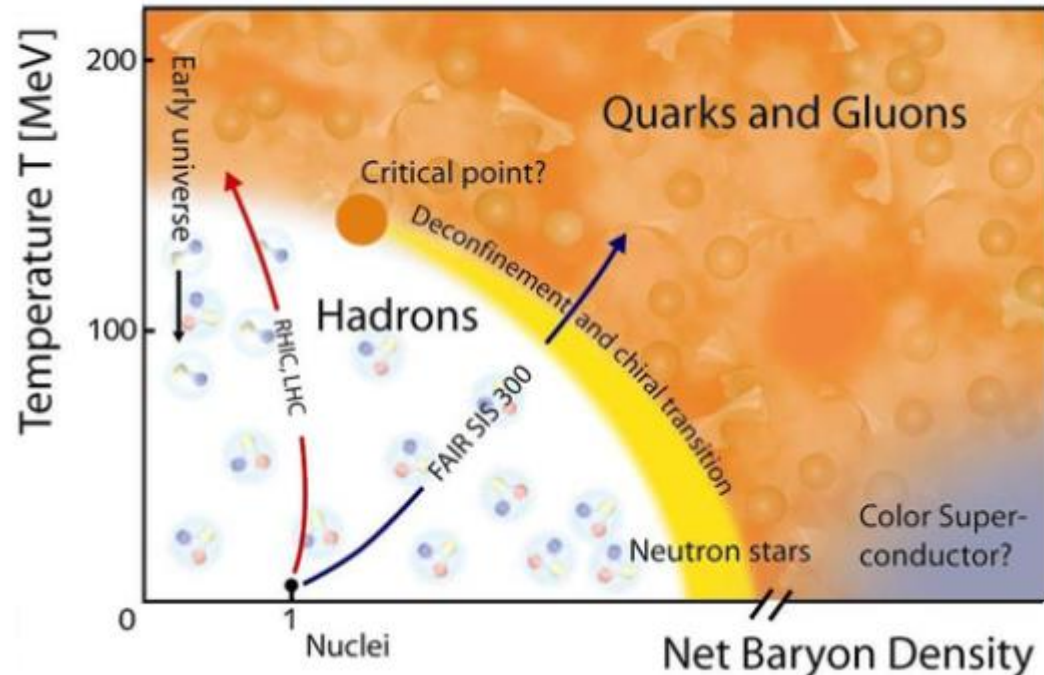
LHC and its experiments upgrades



The QCD phase transition

The QCD phase transition

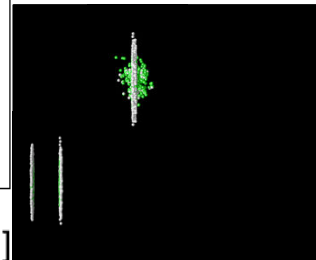
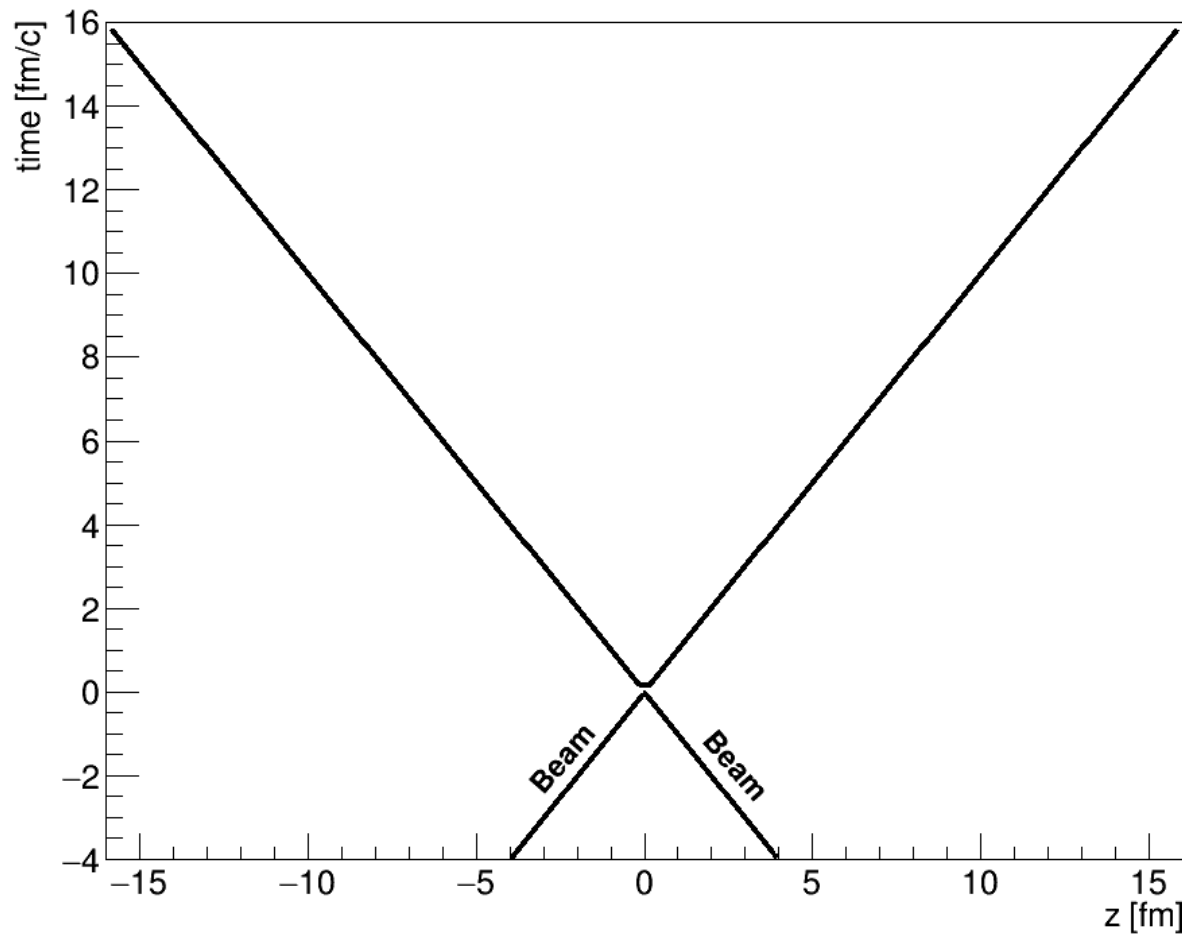
- At LHC net baryon Density ~ 0
- Crossover for physical quark masses
- Confinement and chiral transitions both at $T \sim 155 \text{ MeV}$



Heavy Ion collisions study strongly interacting matter at finite temperature

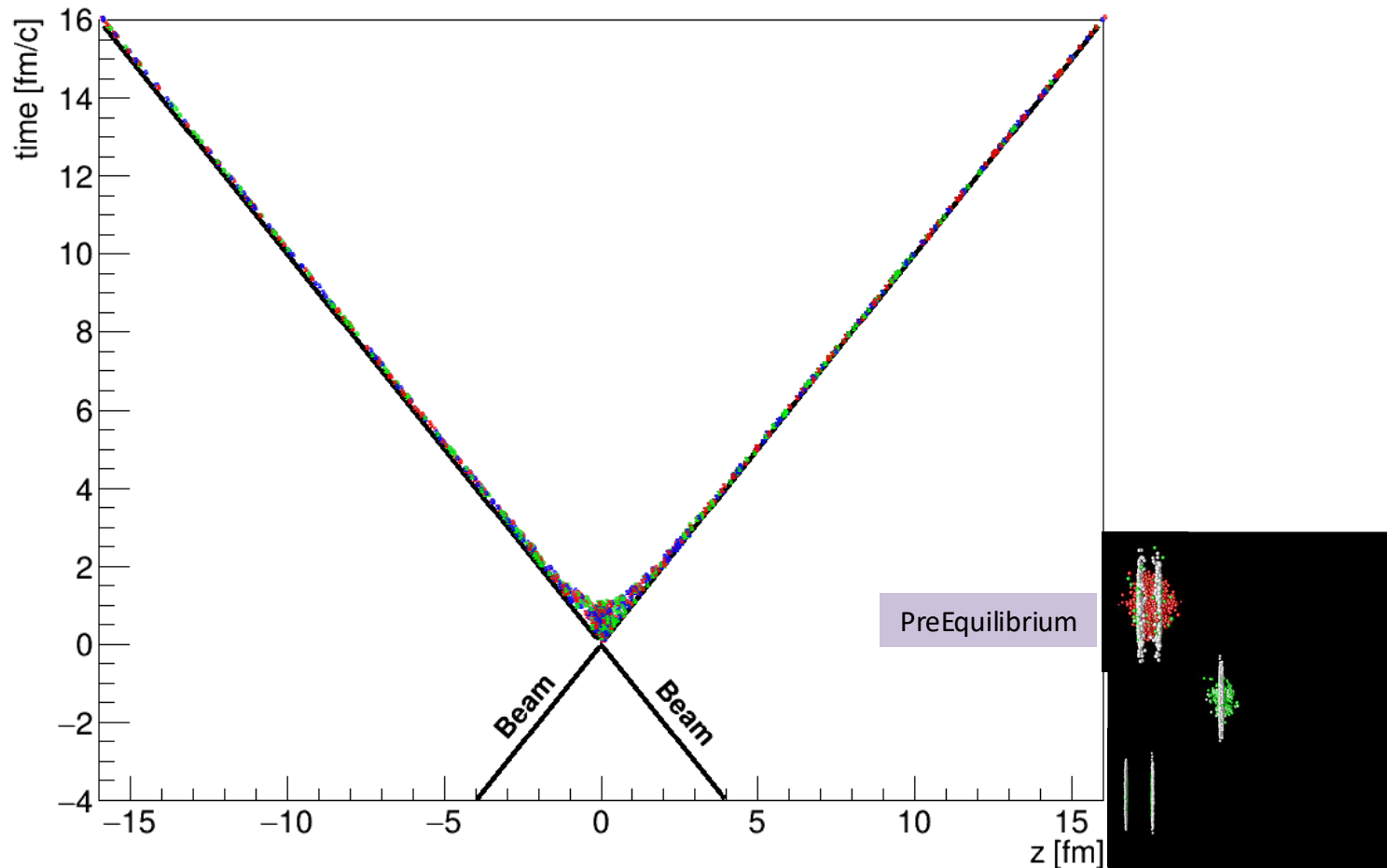
Heavy ion collision

Colliding nuclei



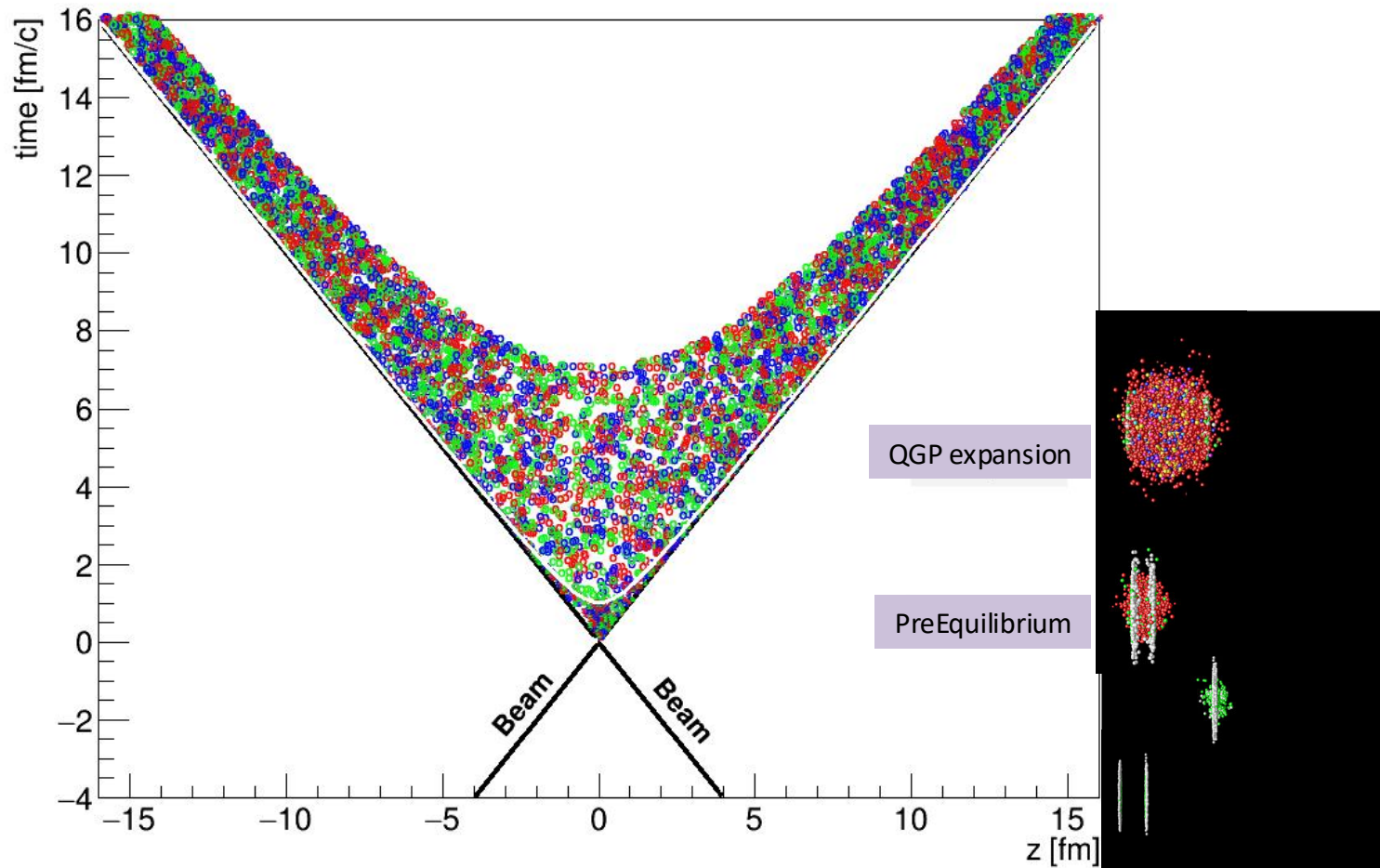
Heavy ion collision

Production of colour medium and equilibration



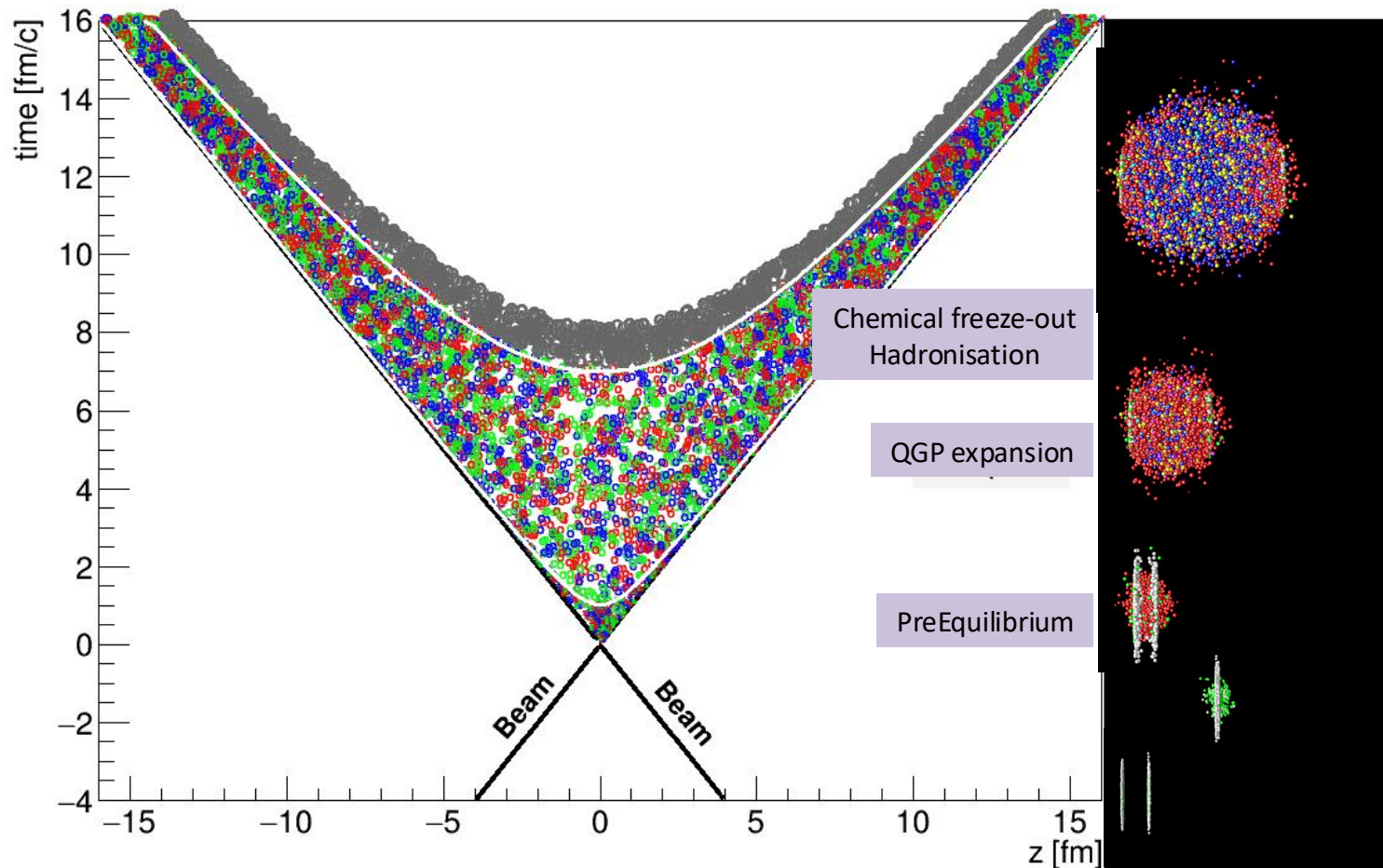
Heavy ion collision

QGP and expansion



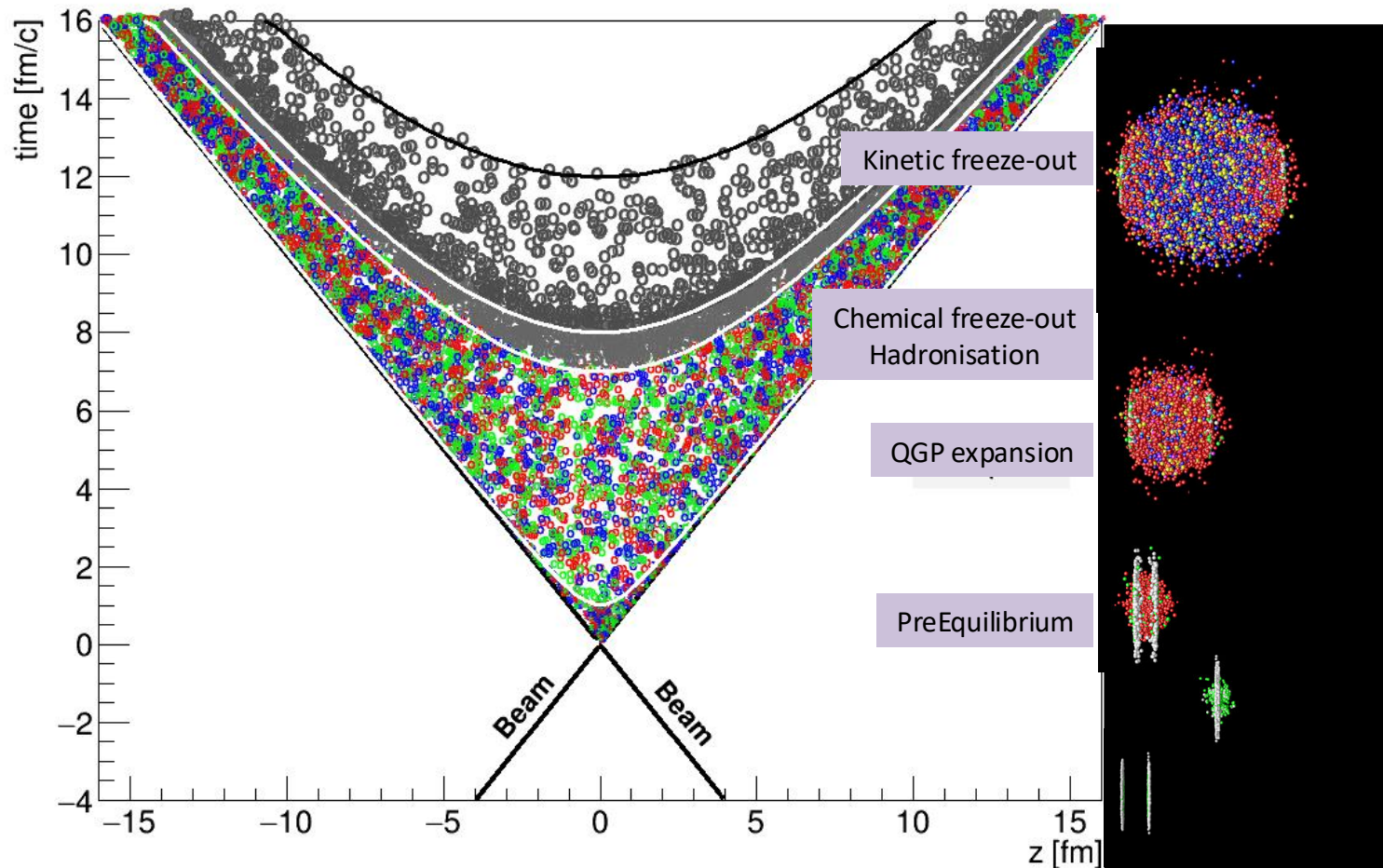
Heavy ion collision

Hadronisation and Chemical freeze-out



Heavy ion collision

Kinetic freeze-out



Heavy ion collisions

■ Observables:

- Hard probes
- Soft probes

■ HI standard model:

- Initial state
 - QCD model/approximation
- Hydrodynamical expansion
 - Ideal or viscous
- Hadronisation
- Hadron transport model

■ Experiments:

– ALICE

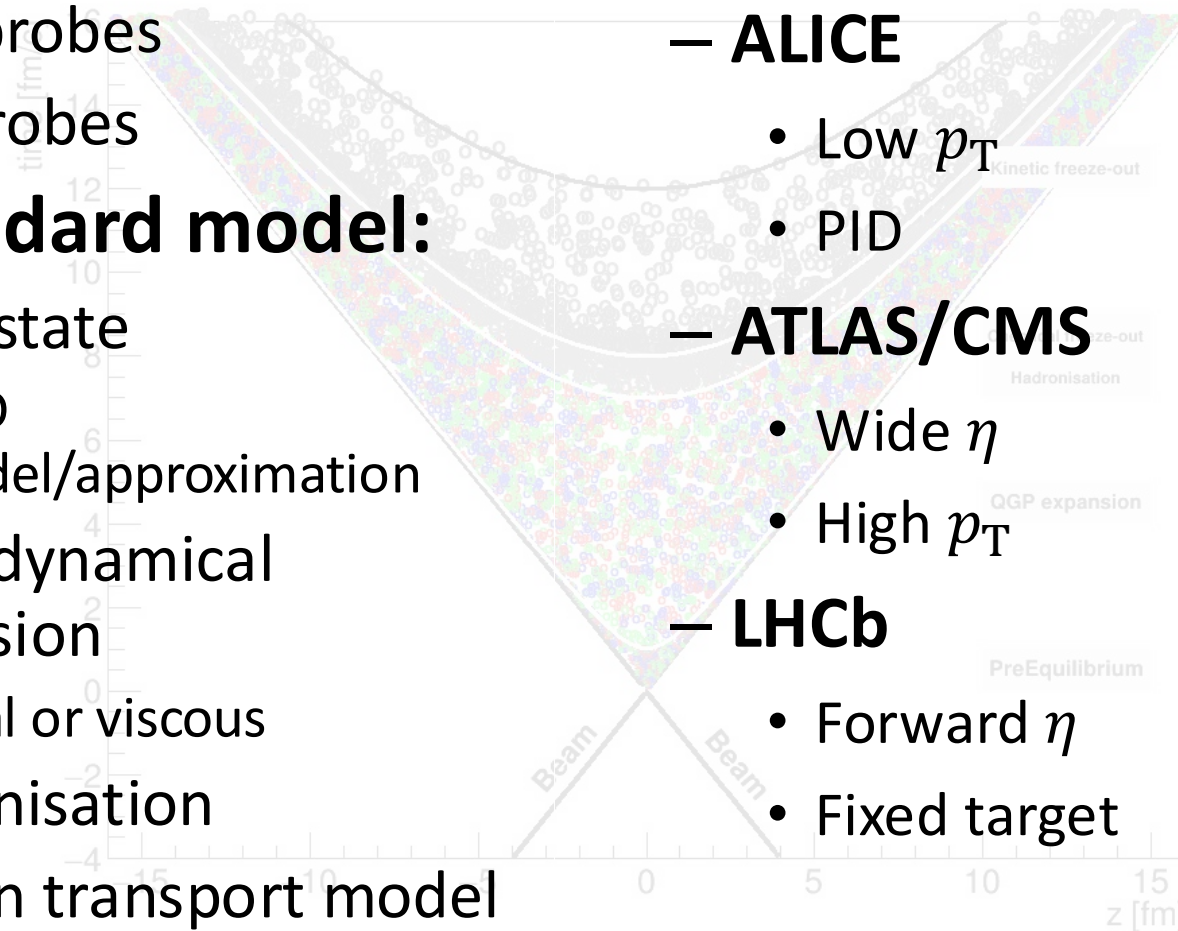
- Low p_T
- PID

– ATLAS/CMS

- Wide η
- High p_T

– LHCb

- Forward η
- Fixed target



SM of HI

- Preequilibrium - Initial state for hydro
 - Glauber
 - Minijet + saturation (EKRT)
 - Colour glass condensate
 - Kharzeev-Levin-Nardi (KLN)
 - IP Glasma
 - TRENTO - parametric description
- Hydro (viscous)
- Hadronisation
- Hadron gas evolution

Hard probes

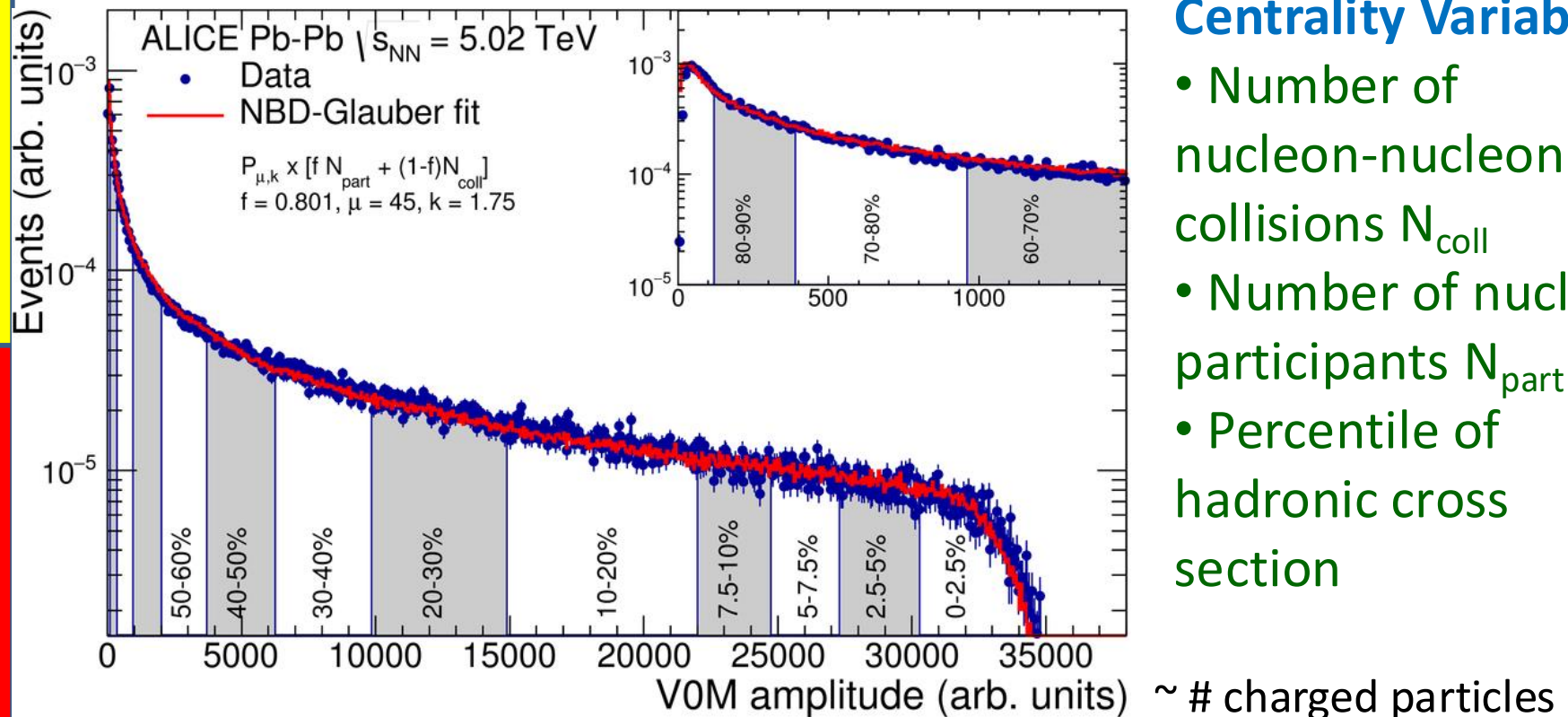
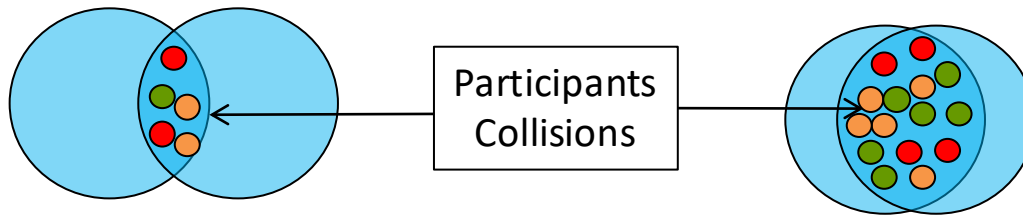
- $p_T \gg$ QCD natural scale
- QCD factorisation:
 $\sigma(AA \rightarrow X) = \text{PDFs} \otimes \sigma(\text{pQCD}) \otimes \text{Fragmentation Functions}$
- Compare pp – pPb – AA
- **Not discussed in this talk**

Soft Observables

- Multiplicity and Transverse energy
- Spectra and flow
- Particle yields – chemistry
- EoS – speed of sound
- Conserved quantum number fluctuations
- Spectral function
- Small system collectivity

Geometry of heavy ion collisions

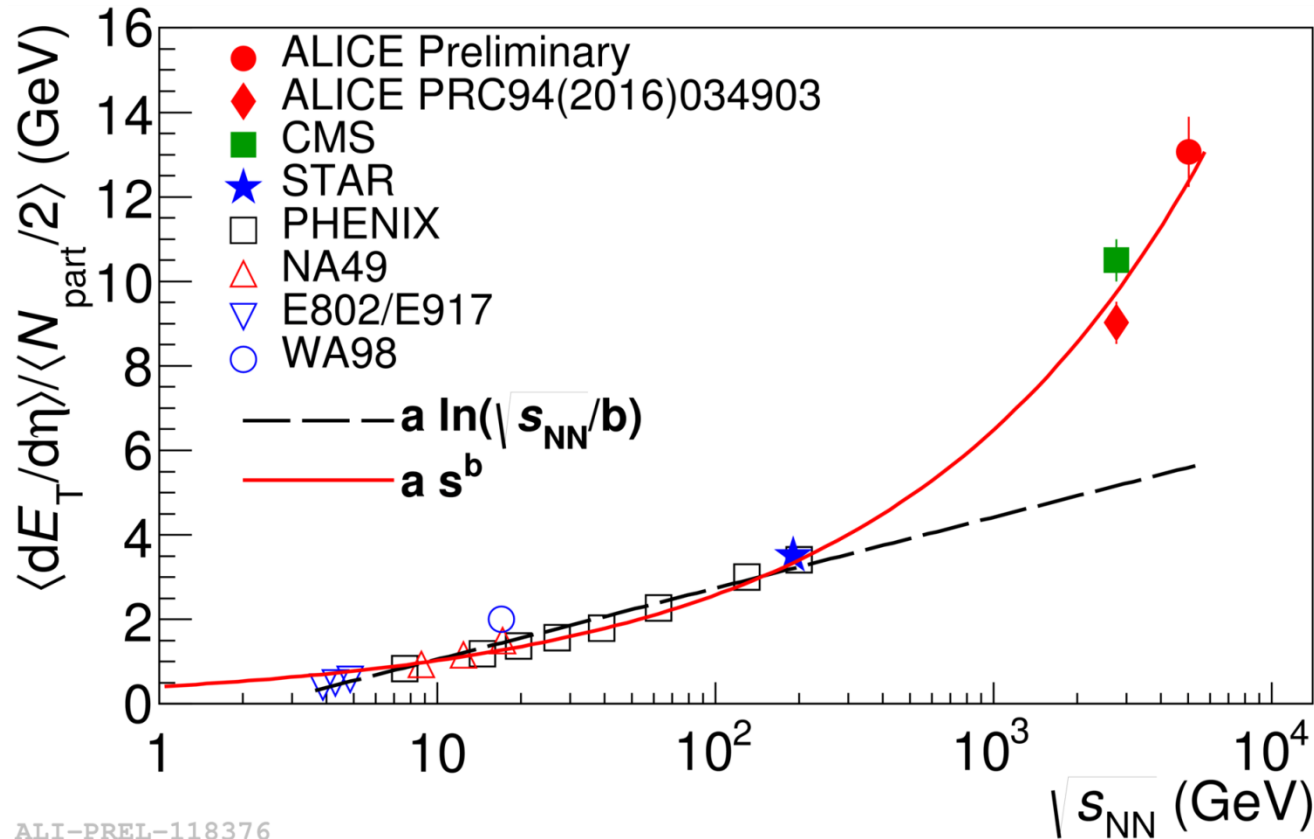
We can control (a posteriori) the geometry of heavy ion collisions



Centrality Variables:

- Number of nucleon-nucleon collisions N_{coll}
- Number of nucleon participants N_{part}
- Percentile of hadronic cross section

Transverse energy – *Energy density*



ALI-PREL-118376

Produced particles:

$$dE_T/d\eta = (2016.5 \pm 5.7 \pm 144.3) \text{ GeV}$$

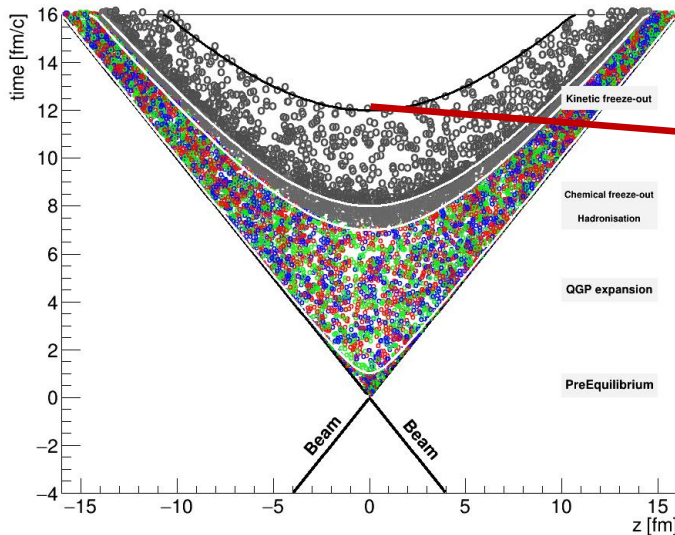
Energy density (Bjorken 1983):

$$\varepsilon \sim 10 \text{ GeV/fm}^3, T \sim 300 \text{ MeV at } \tau_0 = 1 \text{ fm/c}$$

$$\varepsilon(\tau) = \frac{E}{V} = \frac{1}{\tau_0 A} \frac{dE_T}{dy}$$

Transverse momentum spectra

Kinetic freeze-out temperature and expansion velocity



Hadron spectra:

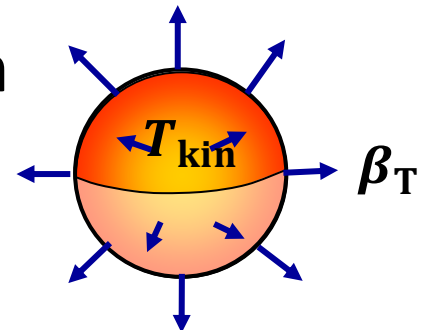
$$\frac{dN}{dydp_T} = \int \frac{d^3N}{dp_T dy d\varphi} d\varphi$$

p_T – transverse momentum
 y – rapidity
 φ – azimuthal angle

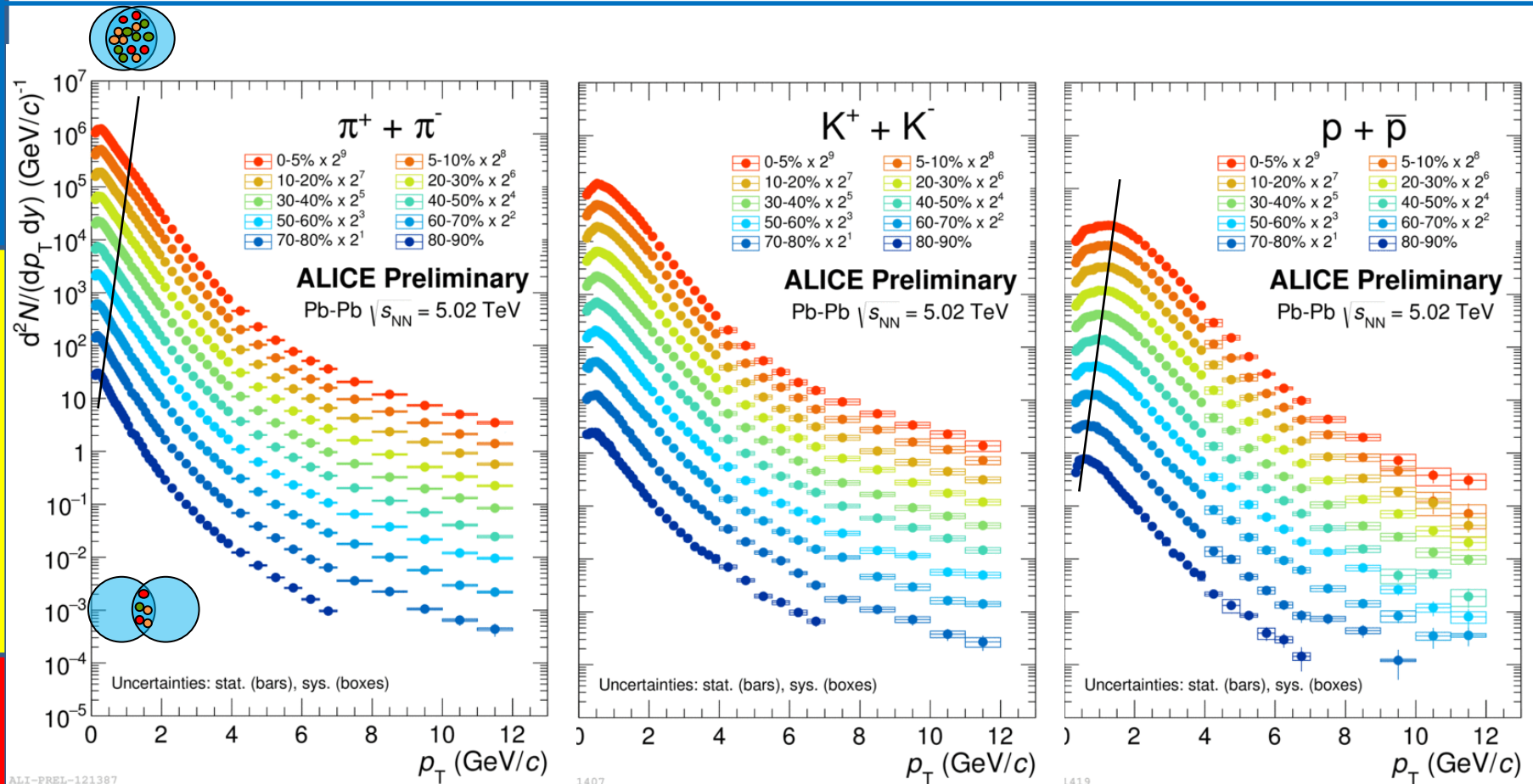
Model:

Blast Wave – hydro inspired parametrisation

- Kinetic freeze-out temperature T_{kin}
- Transverse expansion velocity β_T



Hadron spectra Pb-Pb

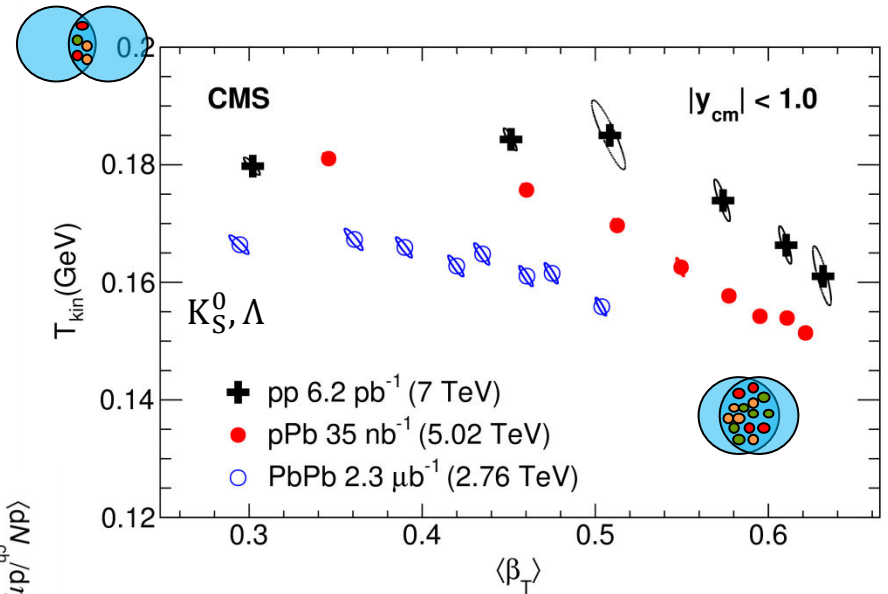
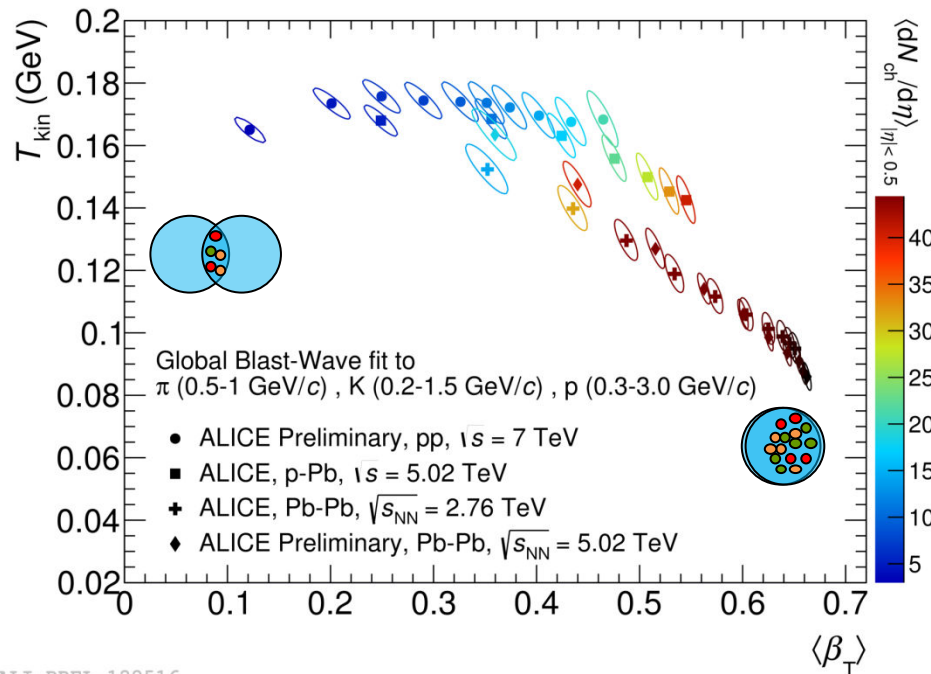
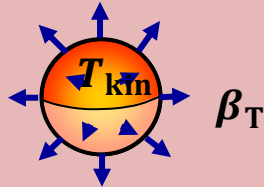


Hardening of spectra as expected in hydro expansion

- With centrality
- With the particle mass $m_T \rightarrow m_T + m_0 \gamma \beta_T$

Kinetic freeze-out

- Temperature versus expansion velocity for pp, p-Pb, Pb-Pb

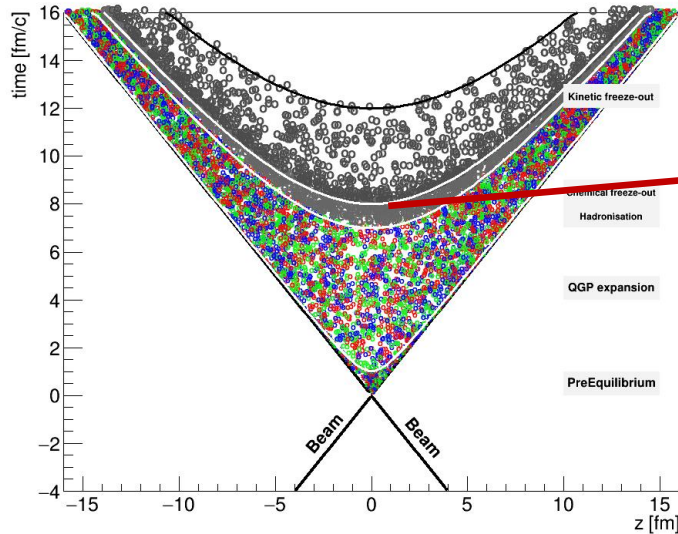


PLB 768 (2017) 103-129

- Similar trend for Pb-Pb, p-Pb and pp

Particle yields

Chemical freeze-out temperature



Hadron yield:

$$Y = \frac{dN}{dy} = \int \frac{d^3N}{dp_T dy d\phi} d\phi dp_T$$

p_T – transverse momentum

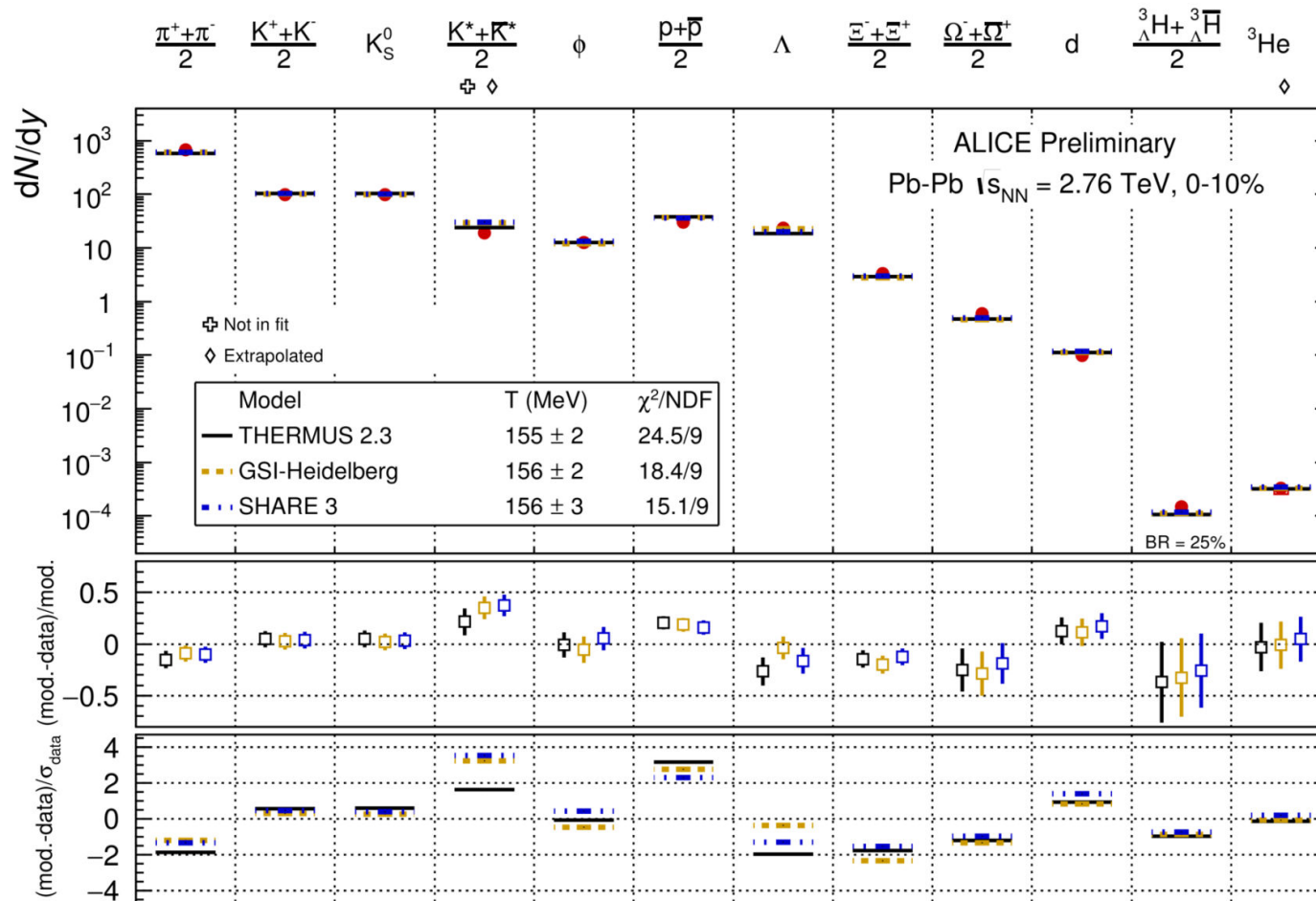
y – rapidity

ϕ – azimuthal angle

Models

- Thermal models – Ideal Hadron Resonance Gas (HRG)
- EPOS, PYTHIA, ...

Thermal model



ALI-PREL-94600

THERMUS: Wheaton et al, Comput. Phys. Commun., 180 84

GSI-Heidelberg: Andronic et al, PLB 673, 142

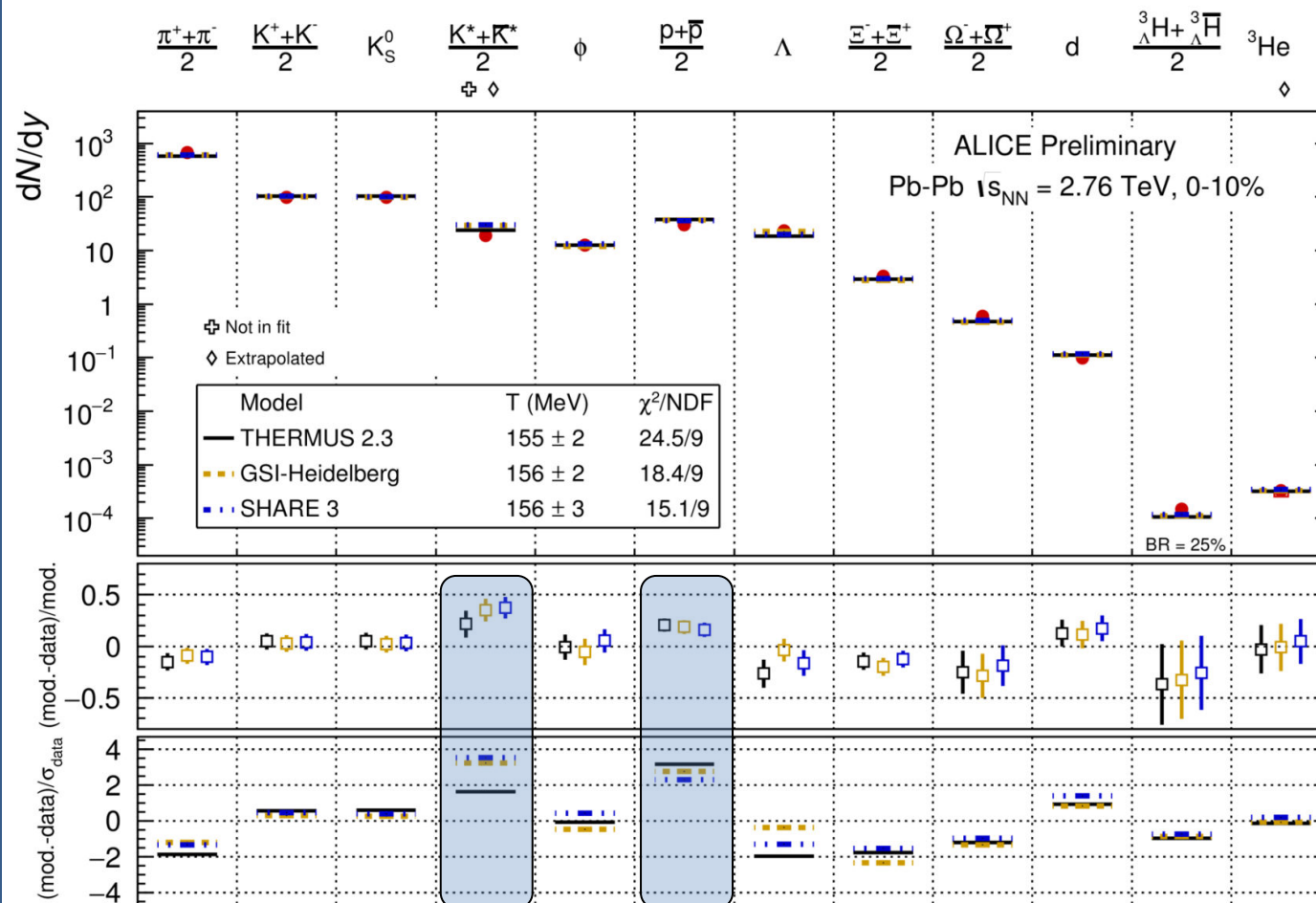
SHARE: Petran et al, Comput. Phys. Commun., 185 Issue 7, 2056

[Fermi 1950](#)

- Describes hadron yields assuming chemical equilibrium
- $T \sim 156$ MeV
 $\sim$ lattice QCD phase transition
 $==$

[ALICE paper](#)

Thermal model



- Describes hadron yields assuming chemical equilibrium
- $T \sim 156$ MeV
~ lattice QCD phase transition
- Deviations for:
 - K^{*0} sequential freeze-out ?
 - p non ideal HRG ?

ALI-PREL-94600

THERMUS: Wheaton et al, Comput. Phys. Commun., 180 84

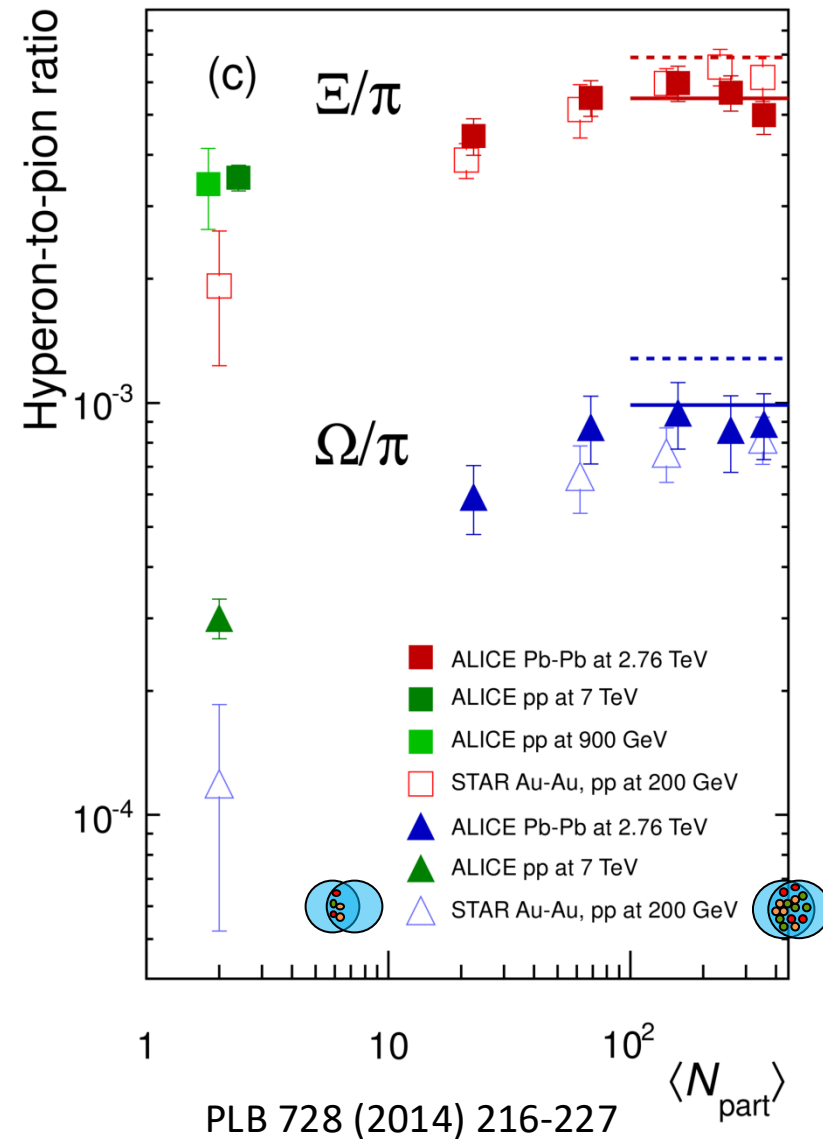
GSI-Heidelberg: Andronic et al, PLB 673, 142

SHARE: Petran et al, Comput. Phys. Commun., 185 Issue 7, 2056

Strangeness enhancement

- Chiral symmetry restored in QGP – strangeness reach chemical equilibrium
- Enhancement = $\frac{Y(H)}{Y(\pi)}$;
 $H = \Lambda, \Xi, \Omega$
- Thermal models for $N_{part} > 150$ describe saturated ratio at $T \sim 165 \text{ MeV}$

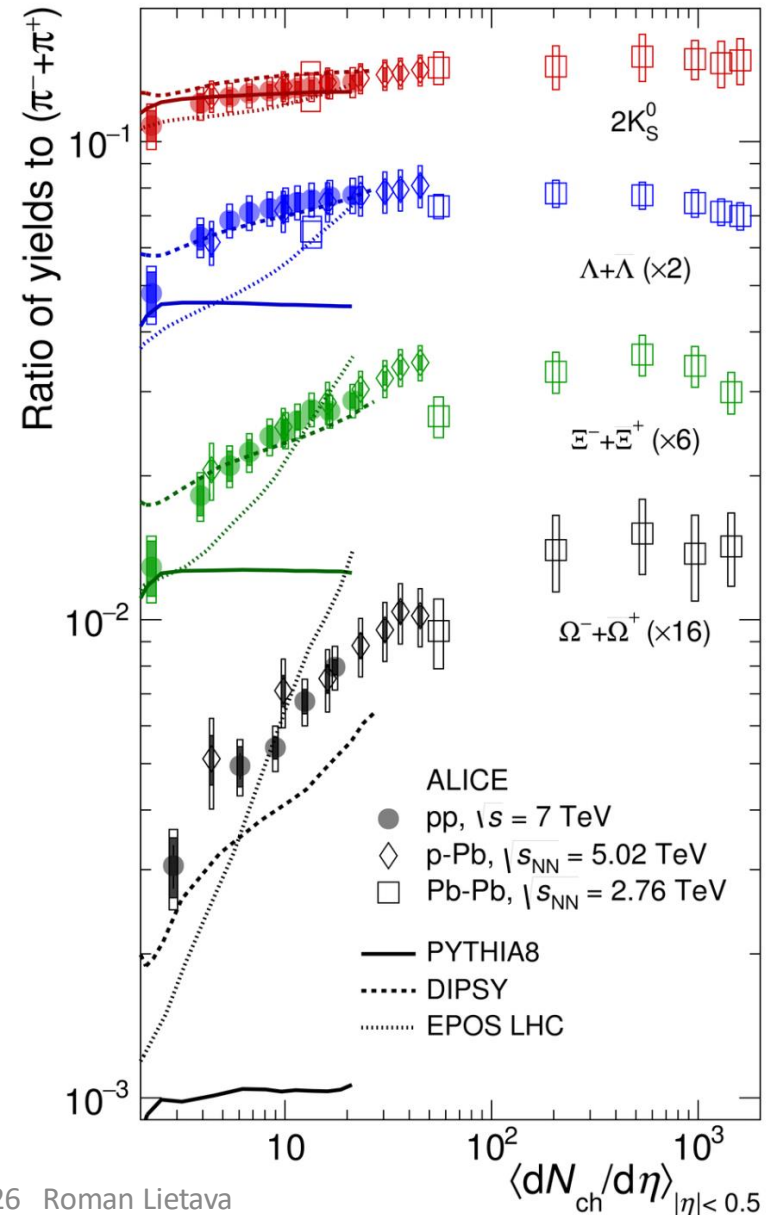
Rafelski/Muller PRL48 106 1982



ALI-PUB-78357

Strangeness enhancement

- Enhancement for pp, p-Pb and Pb-Pb in multiplicity classes
- Smooth evolution from pp/p-Pb to peripheral Pb-Pb collisions
- Scaling with $dN/d\eta$
- Models fail to describe



NATURE PHYSICS | LETTER **OPEN**

Enhanced production of multi-strange hadrons in high-multiplicity proton–proton collisions

ALICE Collaboration

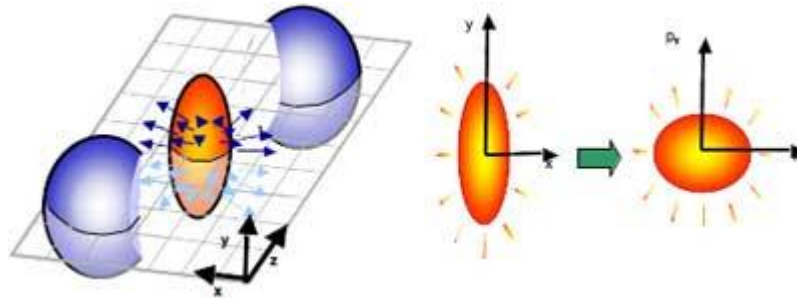
[Affiliations](#) | [Contributions](#) | [Corresponding author](#)

Nature Physics (2017) | doi:10.1038/nphys4111

Received 09 January 2017 | Accepted 23 March 2017 | Published online 24 April 2017

Collective expansion

Observed particle spectrum is the result of the fireball expansion.



If the system is asymmetric in spatial coordinates, scattering converts it to **anisotropy in momentum space**

$$E \frac{d^3 N}{d^3 p} = \frac{d^2 N}{2\pi p_T dp_T dy} \left\{ 1 + 2 \sum_{n=1}^{\infty} v_n(p_T) \cos[n(\varphi - \psi_n)] \right\}$$

Reaction plane ψ_n , Radial flow,

v_1 – direct flow, v_2 - **elliptic flow**

If nuclei overlap is a smooth almond shape odd harmonic are zero.

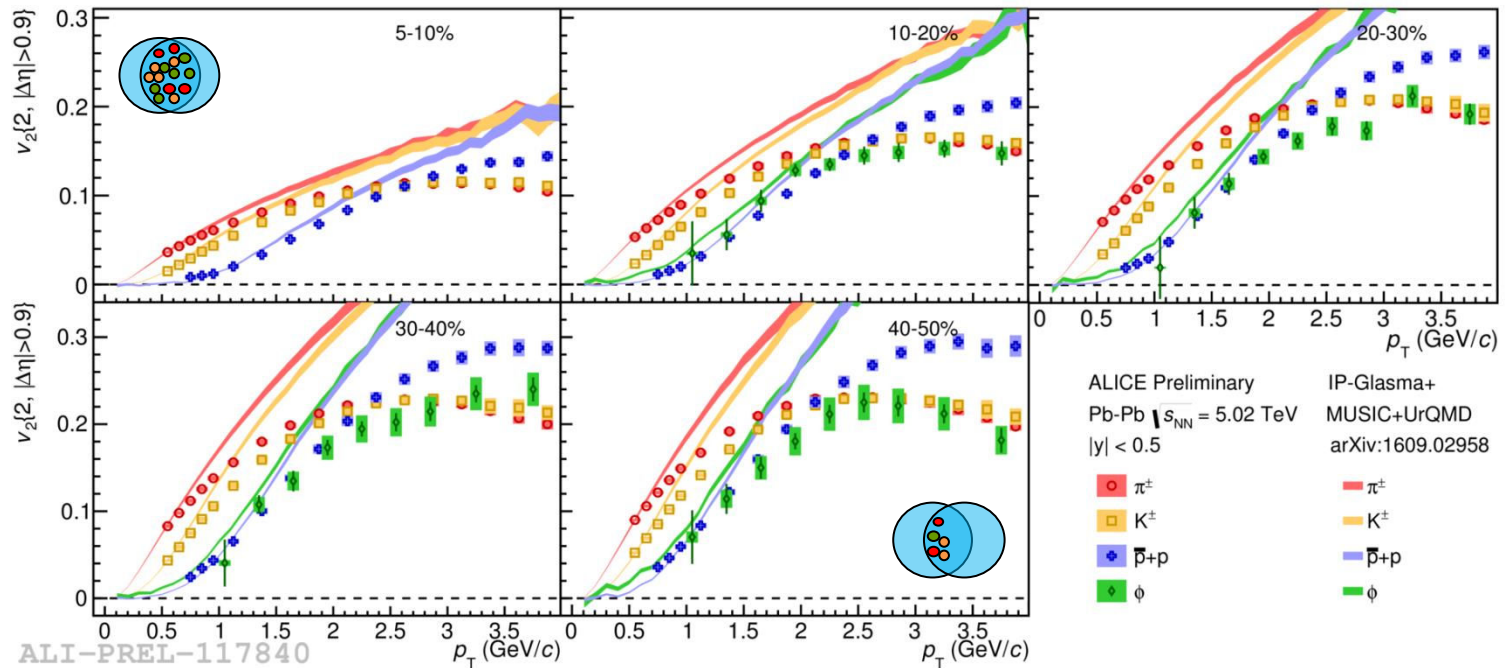
Flow measurement

- Flows v_n measured from particle correlations
- Effects other than collective flow can contribute
 - jet/resonance decays/Bose-Einstein correlations
- Suppress non-flow:
 - Correlate in separated phase space, i.e. pseudorapidity gap
 - Correlate more than 2 particles and subtract lower order
 - technique of **multiparticle cumulants**

$$v_2\{2\} = \sqrt{c_n\{2\}}; \quad c_n\{2\} = \langle\langle 2 \rangle\rangle = \langle\langle \cos n(\varphi_1 - \varphi_2) \rangle\rangle$$

$$v_n\{4\} = \sqrt[4]{-c_n\{4\}}; \quad c_n\{4\} = \langle\langle 4 \rangle\rangle - 2\langle\langle 2 \rangle\rangle;$$
$$\langle\langle 4 \rangle\rangle = \langle\langle \cos n(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4) \rangle\rangle$$

Elliptic flow



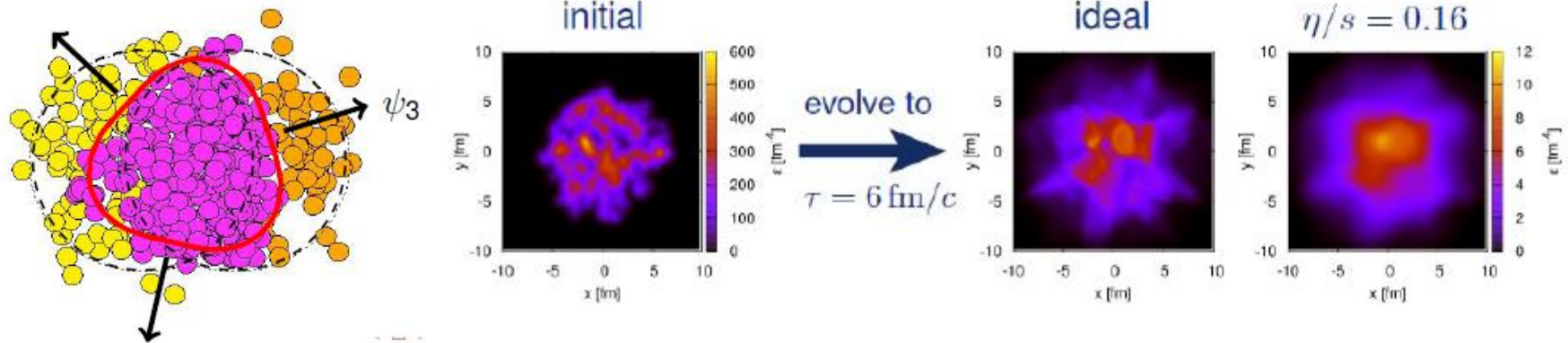
Shear viscosity over entropy density $\eta/s=0.095$ (QM bound 0.08)

- $v_2(p_T)$ in centrality bins compared with hydro prediction (hydro tuned on 2.76 TeV data) [new alice paper](#)
- Hydro describes mass hierarchy
- QGP behaves as almost ideal fluid

Higher harmonics

- **Initial geometry not described by the ideal almond shape**
 - Fluctuations of initial energy/pressure distributions lead to “irregular” shapes that fluctuate event-by-event
- **Higher harmonics more sensitive to the value of shear viscosity**

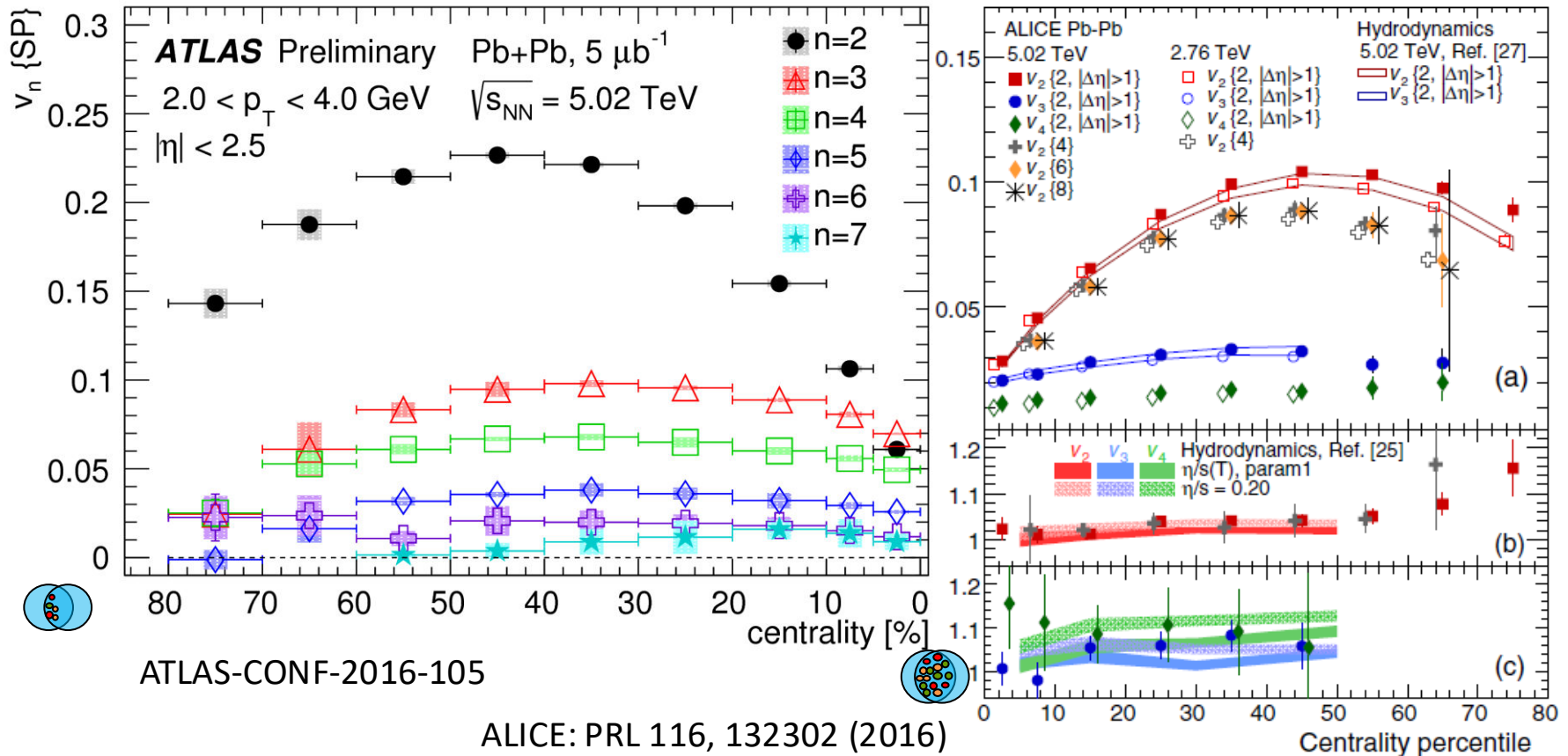
arXiv:1209.6330



Hydro evolution of initial state (ideal and viscous hydro):
fluctuations of initial state are damped by viscosity.

Higher harmonics

Initial geometry not described by the ideal almond shape



Hydro evolution of initial state (ideal and viscous hydro):
 fluctuations of initial state are damped by viscosity.
 For viscosity estimate see next slide.

Initial state and viscosity estimate

- **Bayes estimate** using:
 - Multiplicity (yields)
 - p_T spectra
 - Flow coefficients $v_{2,3,4}$
- Parametric Initial state model TRENTO
 - Initial state entropy
 $s(\tau_0) = f(p)$
- Evolution:
 - Viscous hydro
 - Hadron cascade
- 9 parameters

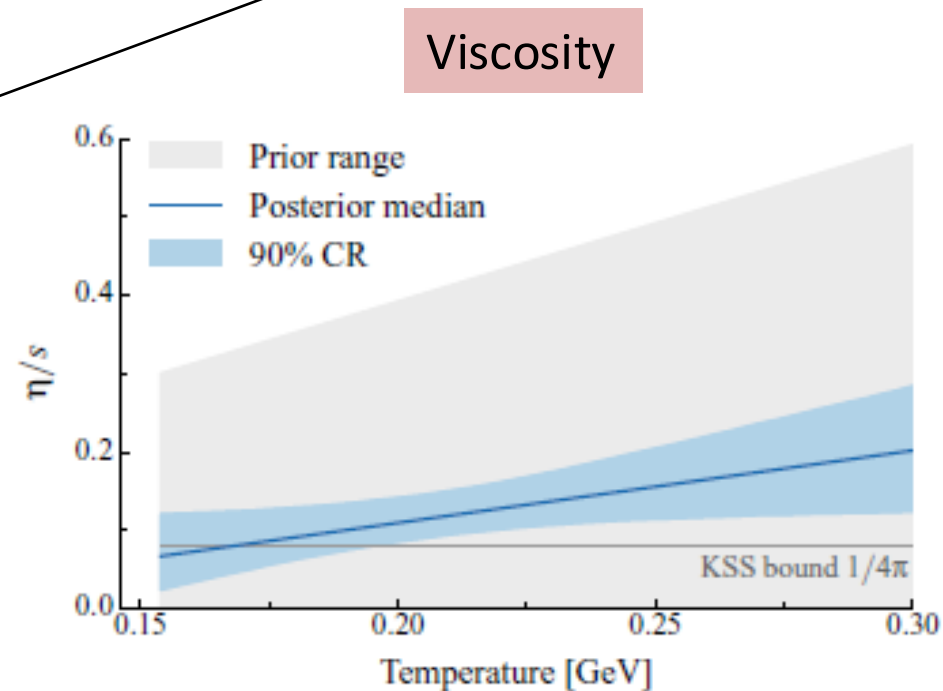
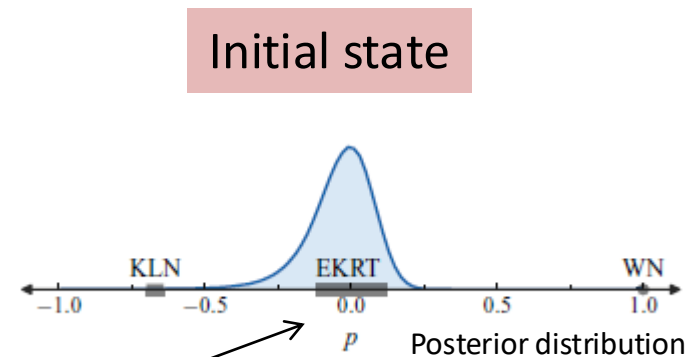
PRC94, 024907 (2016)

Initial state and viscosity estimate

- **Bayes estimate** using:
 - Multiplicity (yields)
 - p_T spectra
 - Flow coefficients $v_{2,3,4}$
- Parametric Initial state model TRENTO
 - Initial state entropy

$$s \propto \left(\frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

- Evolution:
 - Viscous hydro
 - Hadron cascade
- 9 parameters



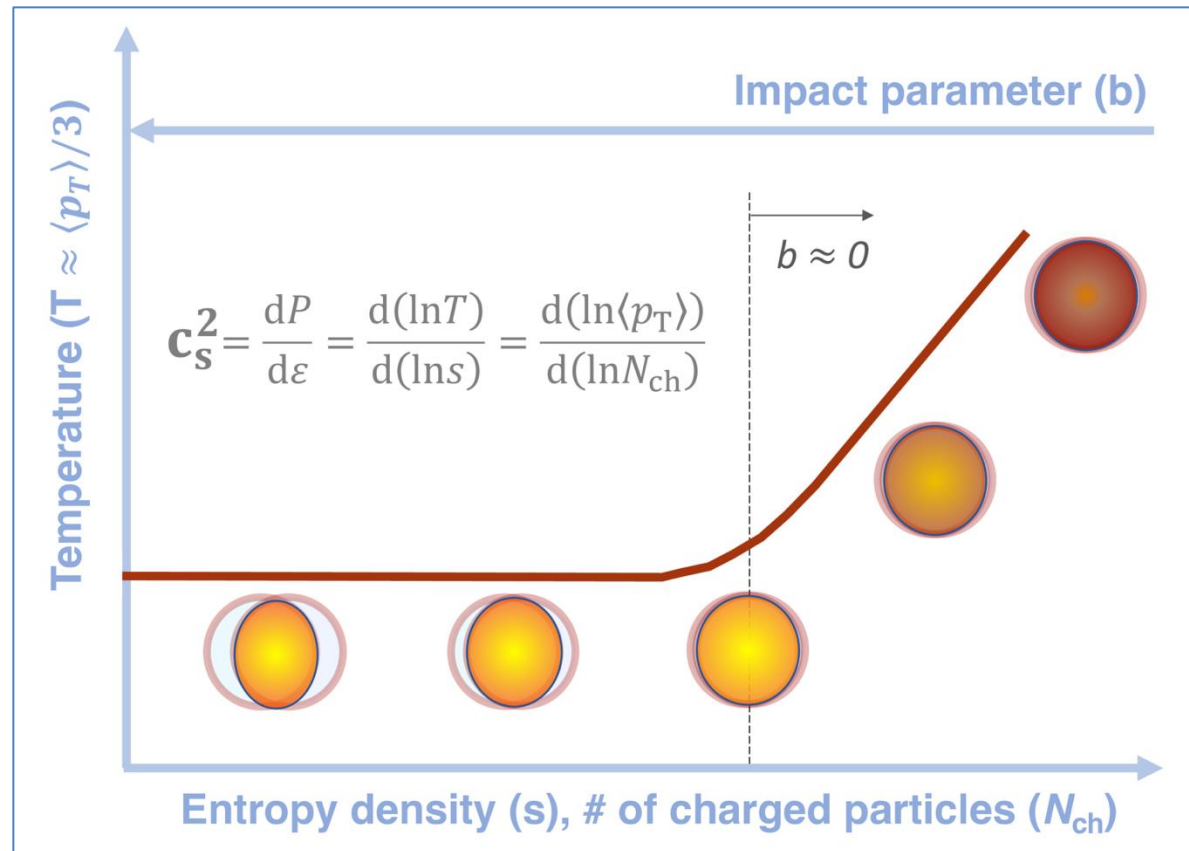
PRC94, 024907 (2016)

Lattice and Heavy ions

- Cross over temperature T_c
- Equation of state $p(\epsilon)$
 - Speed of sound
- Fluctuation of conserved charges (susceptibility)
- Static screening (quarkonia)
- Spectral function
 - Dileptons < 3 GeV
 - quarkonia
 - transport coefficients

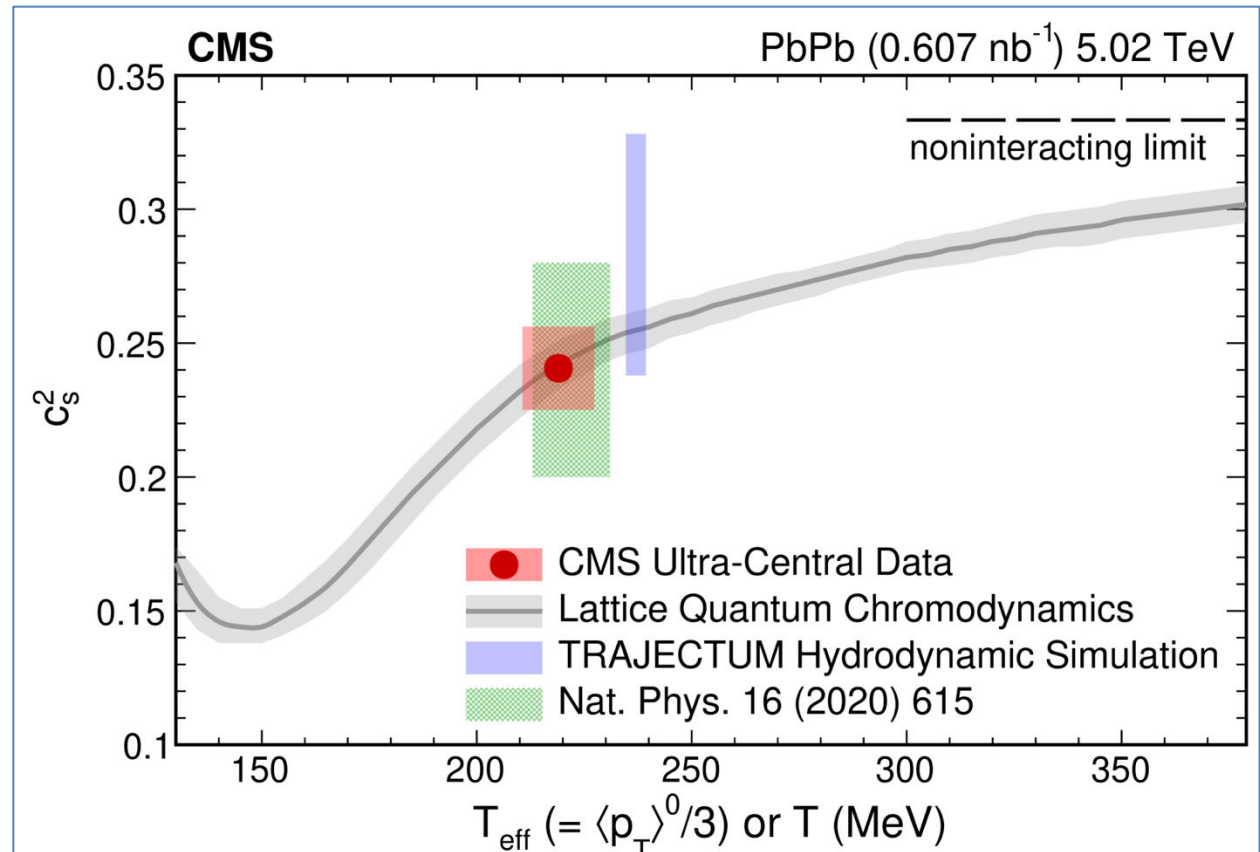
Speed of sound I

- Ollitrault et al proposed and CMS did 'direct' measurement of speed of sound in ultracentral collisions [SOS CMS paper](#)



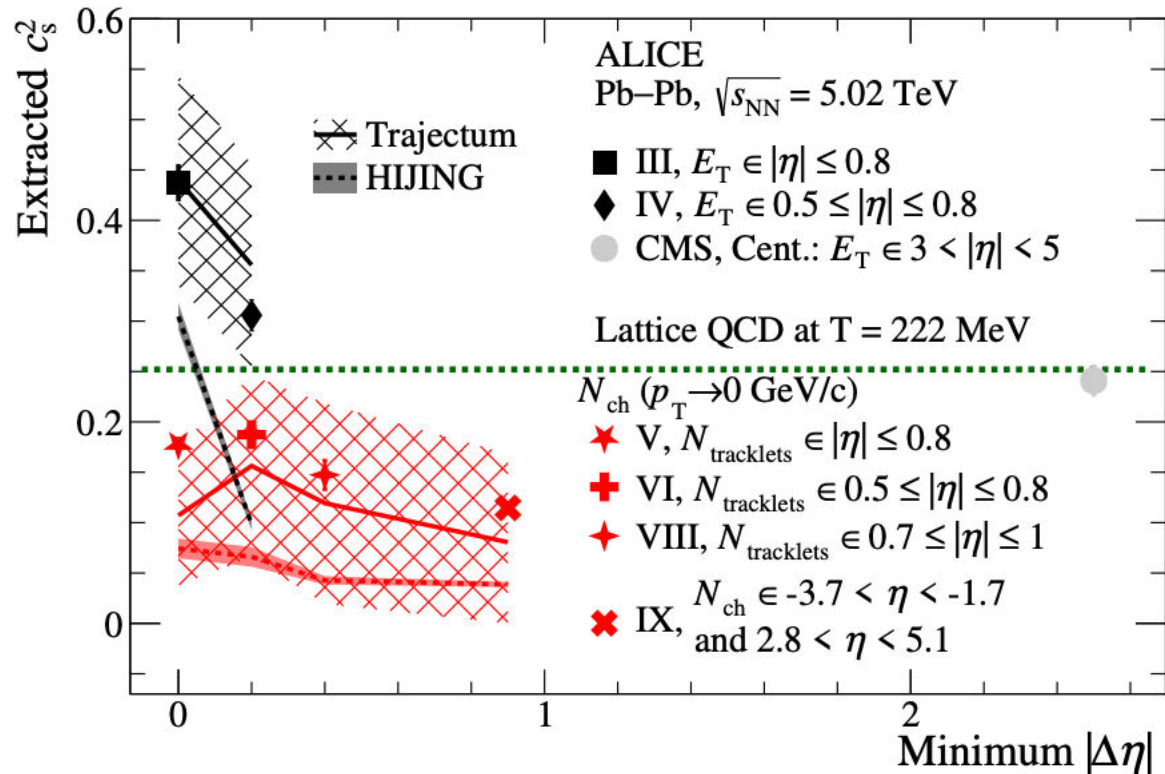
Speed of sound I

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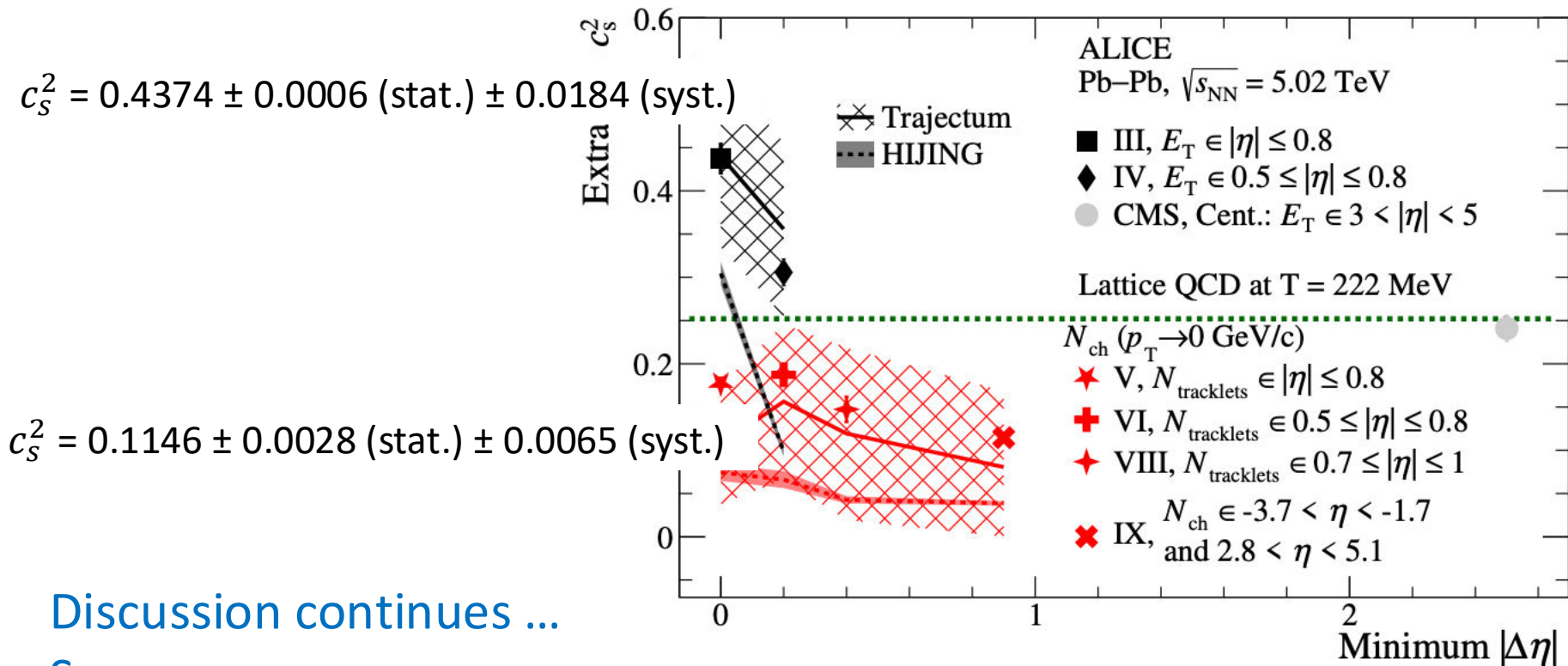
Speed of sound II

- ALICE repeated measurement [SOS ALICE paper](#)
- centrality estimators biases - reduced by using
 - N_{ch} instead of E_T
 - Rapidity gap between $\langle p_T \rangle$ and centrality estimator
- Wide range of extracted c_s



Speed of sound II

- ALICE repeated measurement [SOS ALICE paper](#)
- centrality estimators biases
- Reduced by using
 - N_{ch} instead of E_T
 - Rapidity gap between $\langle p_T \rangle$ and centrality estimator
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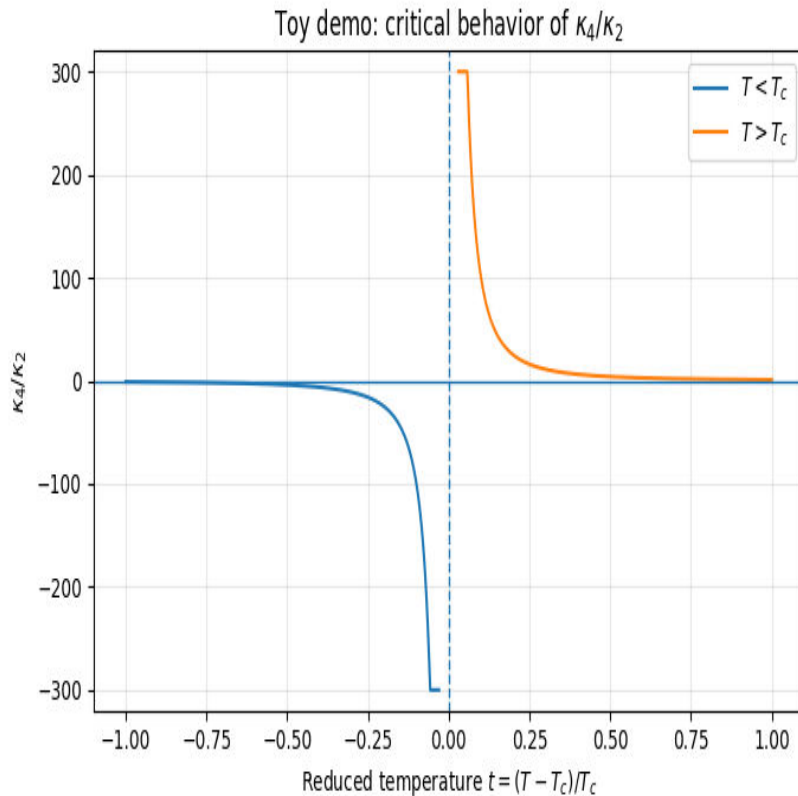
Discussion continues ...

See [Ultracentral heavy ion collisions, transverse momentum and the equation of state](#)

Criticality

- Fluctuation of net conserved charges – susceptibilities
$$\chi_n = \frac{\partial^n (P/T^4)}{\partial (\mu/T)^n}$$
 - Cumulants $\kappa_n = VT^3 \chi_n$
 - Baryon number (B), charge, strangeness
- Experimentally non trivial
 - $\kappa_{measured} = \kappa_{critical} + \kappa_{geom} + \kappa_{other}$
 - Acceptance
 - Volume fluctuations
- Look for non monotonic behaviour of cumulants ratio versus centrality

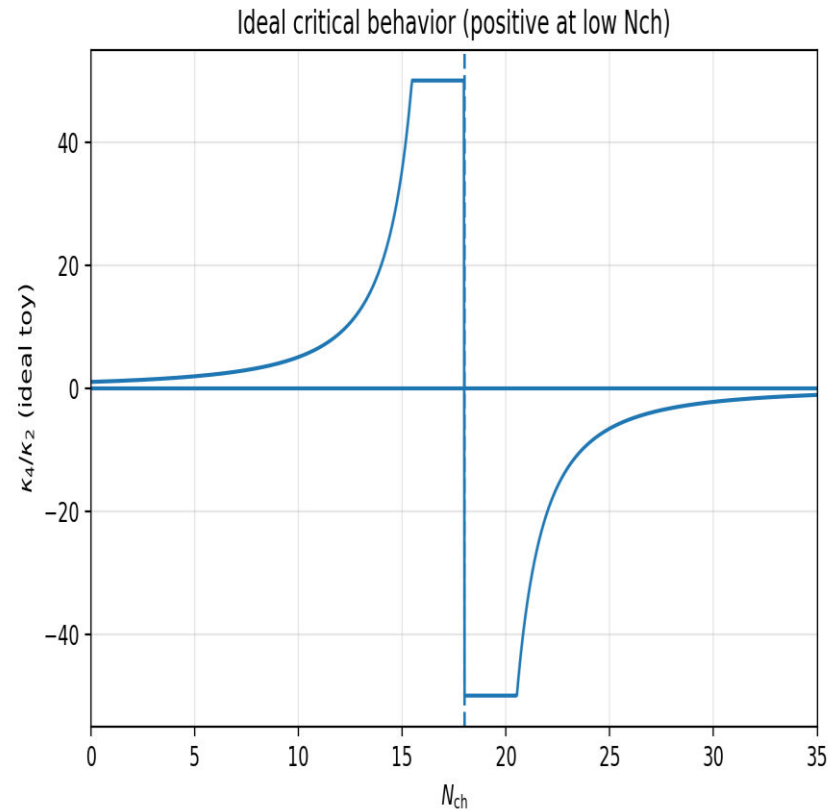
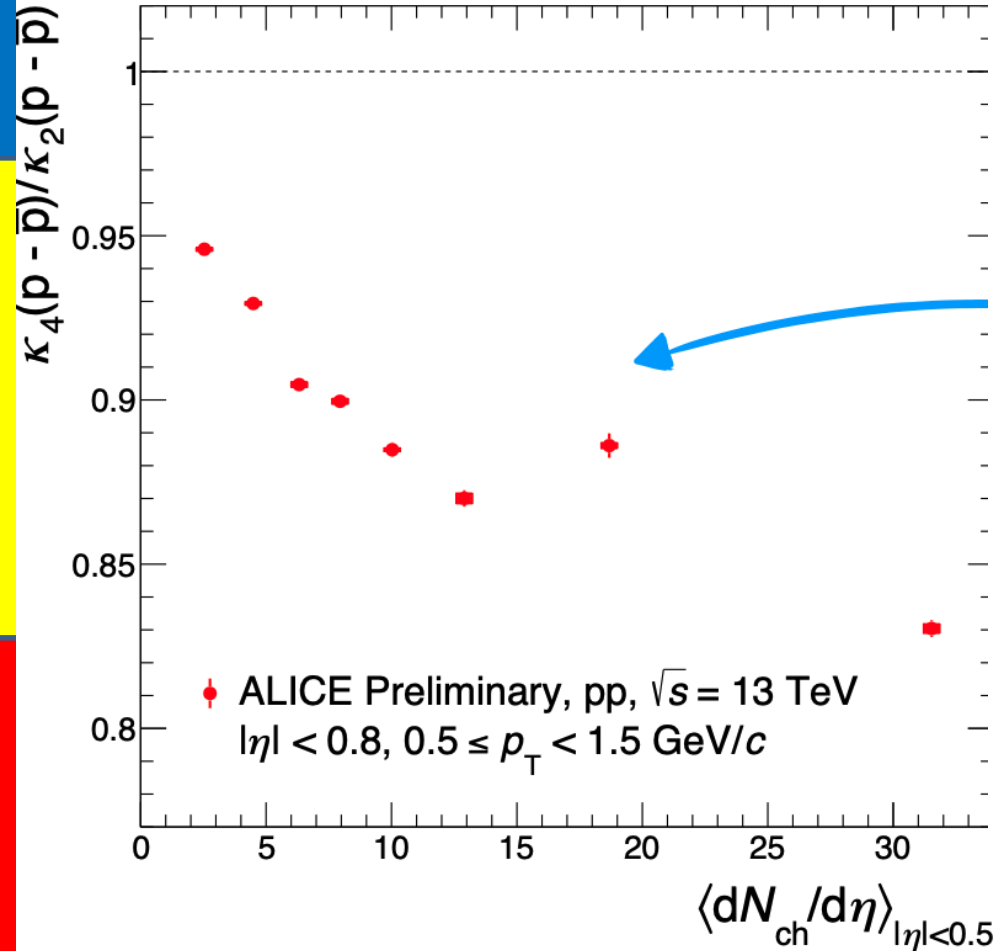
Non monotonic behaviour: Toy



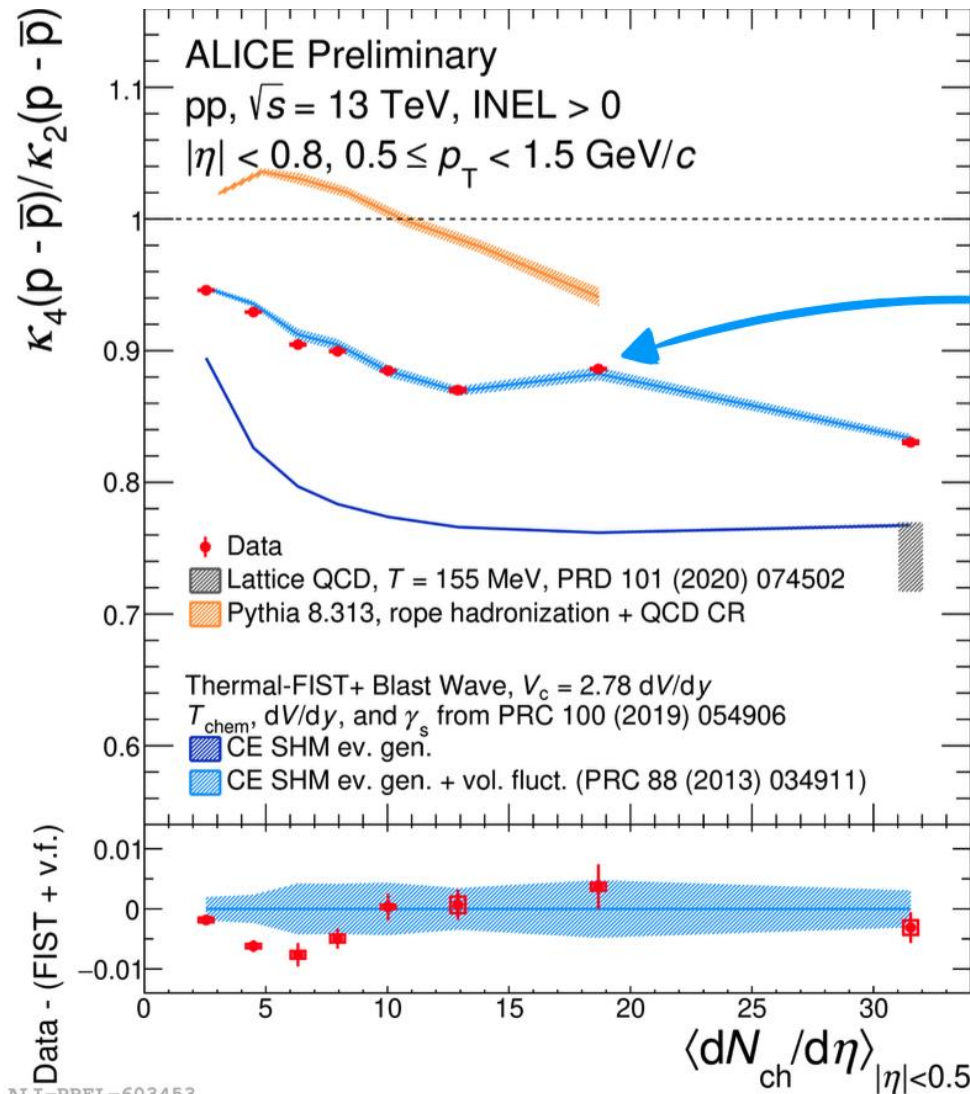
- $\frac{\kappa_4}{\kappa_2} \sim \text{sign}(T - T_c) \frac{1}{|T - T_c|^2}$
- Charged particle multiplicity N_{ch} as proxy to T_c

Non monotonic behaviour

- pp: do we have it ?



Non monotonic behaviour



- pp: do we have it ? **No**
- Correlation between N_{ch} and system size can introduce nonmonotonicity
- => compare data with statistical hadronisation model (SHM) corrected for this effect

pp B cumulants

ALICE: <https://arxiv.org/pdf/2510.10847>

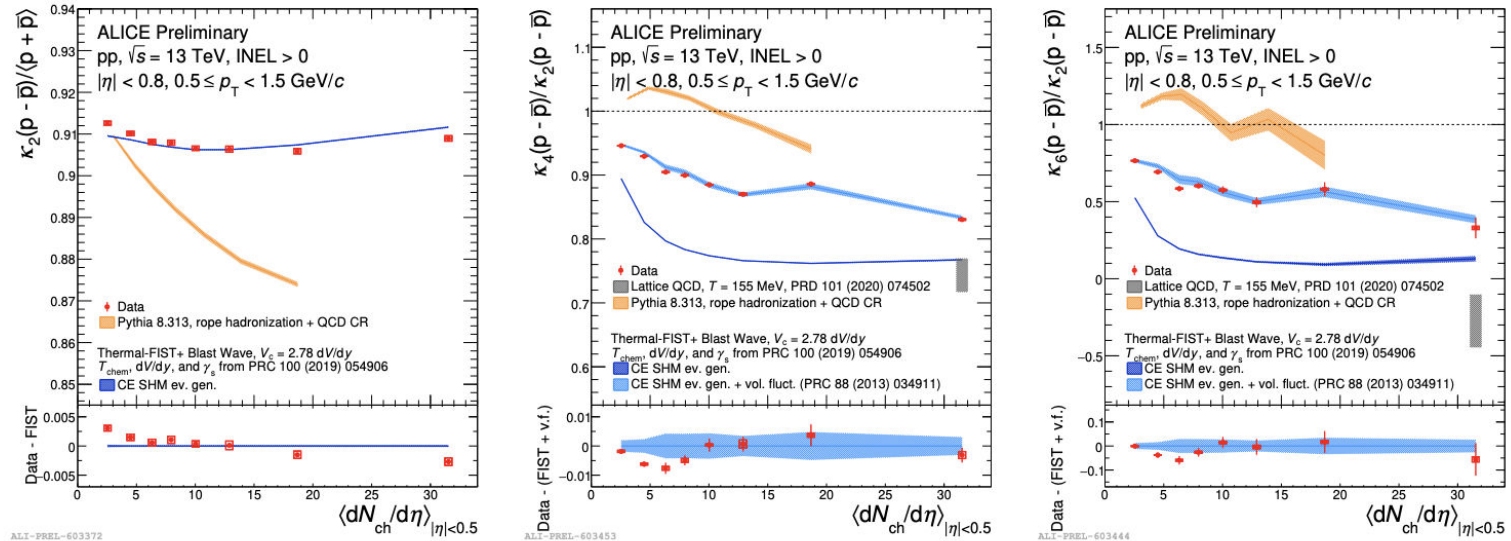


Figure 1: Net-proton number cumulant ratios $\kappa_2(p - \bar{p})/\langle p + \bar{p} \rangle$ (left), $\kappa_4(p - \bar{p})/\kappa_2(p - \bar{p})$ (center) and $\kappa_6(p - \bar{p})/\kappa_2(p - \bar{p})$ (right) in pp collisions at 13 TeV as a function of the charged particle multiplicity density at midrapidity $\langle dN_{ch}/d\eta \rangle_{|\eta|<0.5}$.

- Cumulants well described by statistical model with volume fluctuations
- No sign of criticality

Pb-Pb B cumulants

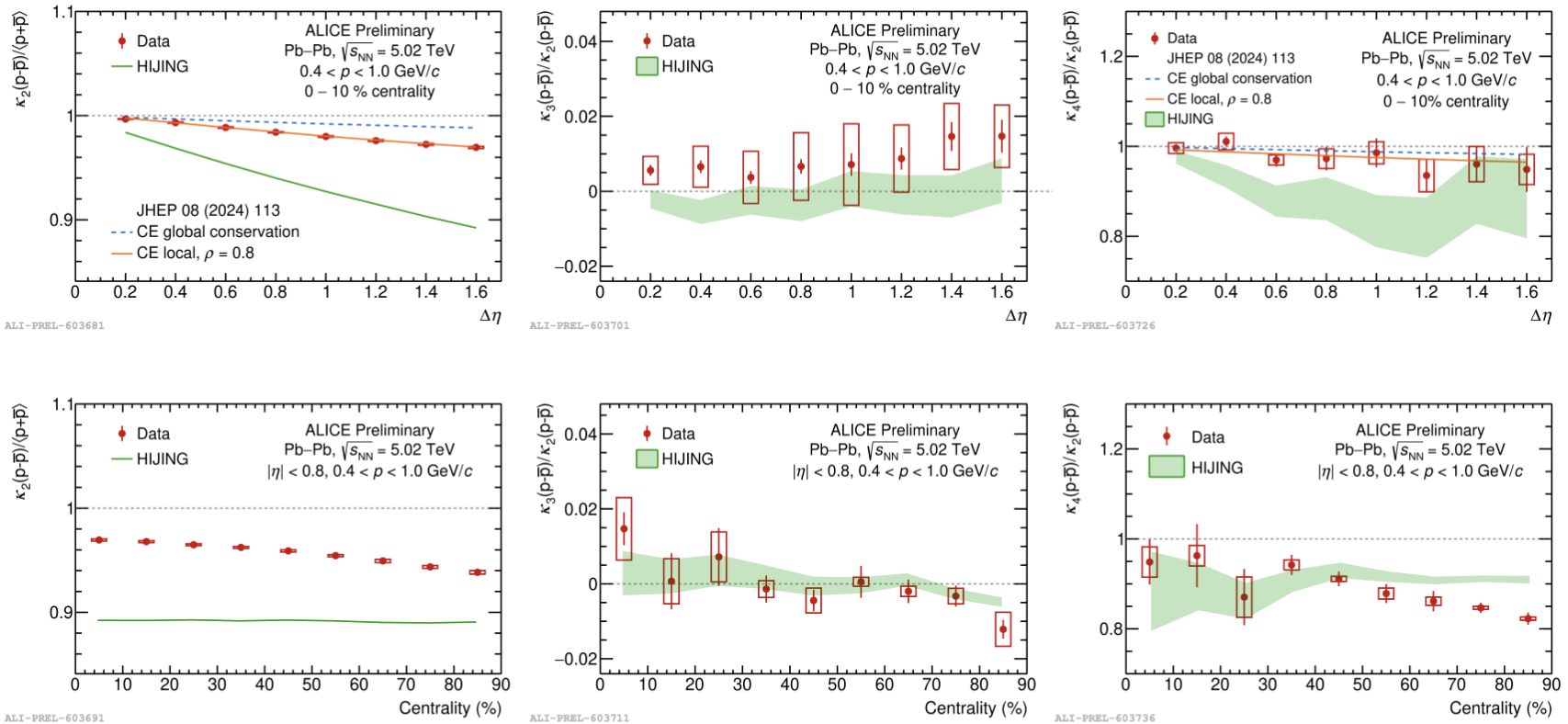


Figure 2: Net-proton number cumulant ratios $\kappa_2/\langle p + \bar{p} \rangle$ (left), κ_3/κ_2 (center) and κ_4/κ_2 (right) in Pb-Pb collisions at 5.02 TeV as a function of the pseudorapidity acceptance $\Delta\eta$ (top) and centrality (bottom).

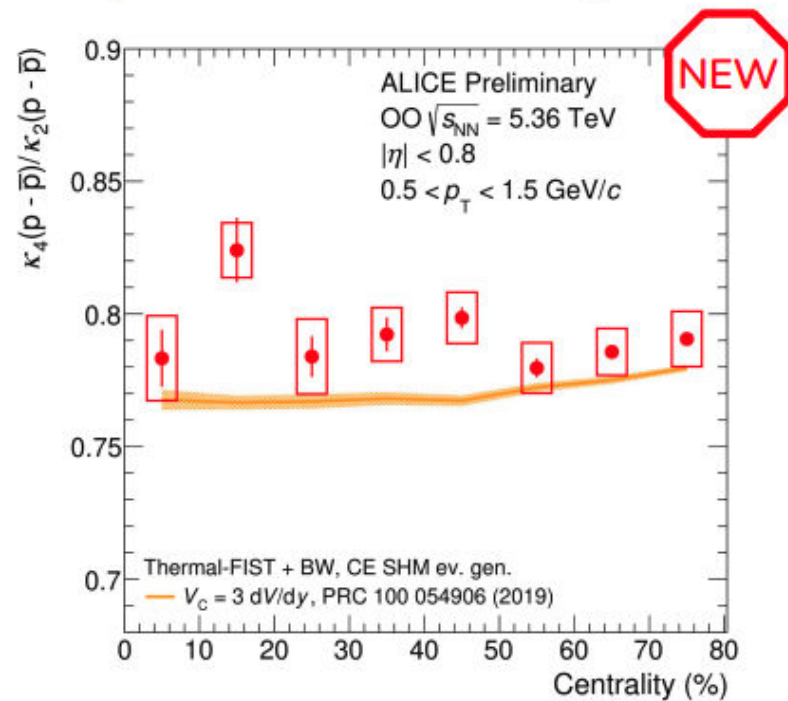
Run3 ALICE - 10 times more data

Run5 ALICE – 50 times more data => higher orders accessible

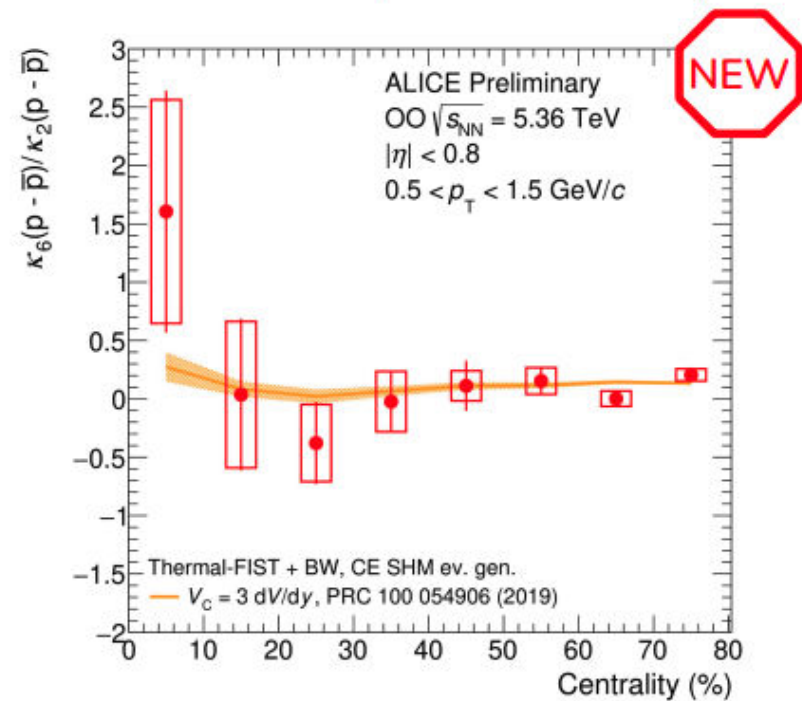
κ_6 in pp versus PbPb

- Why 6th moment (and higher) is more difficult in PbPb than in pp:
- Error $\delta\kappa_6 \simeq (\langle N_p \rangle + \langle N_{\bar{p}} \rangle)^3 \sqrt{\frac{720}{N}}$
- N – number of collected events
- $\Rightarrow$ Error on higher moments grows by power law with system size

OO κ_6



ALI-PREL-623357



ALI-PREL-623362

HRG with local B conservation and volume fluctuation describes OO

Comments on comparison with LQCD

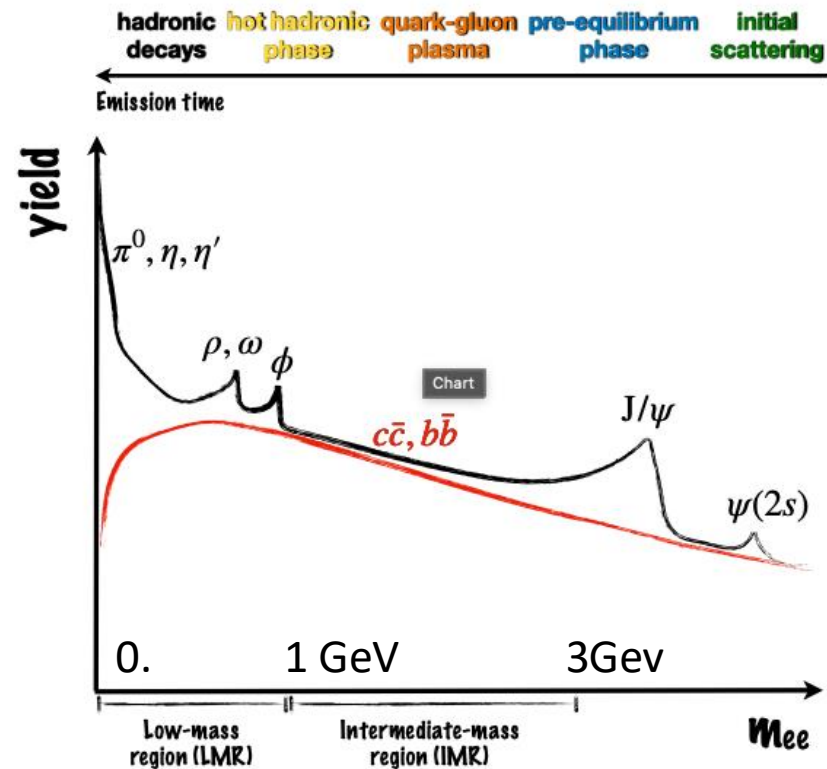
- Max requirements:
 - Net proton fluctuation (not net baryon)
 - Conservation effects:
 - Local Canonical ensemble
- My thanks to Max Puccio for discussion of the talk

Spectral function

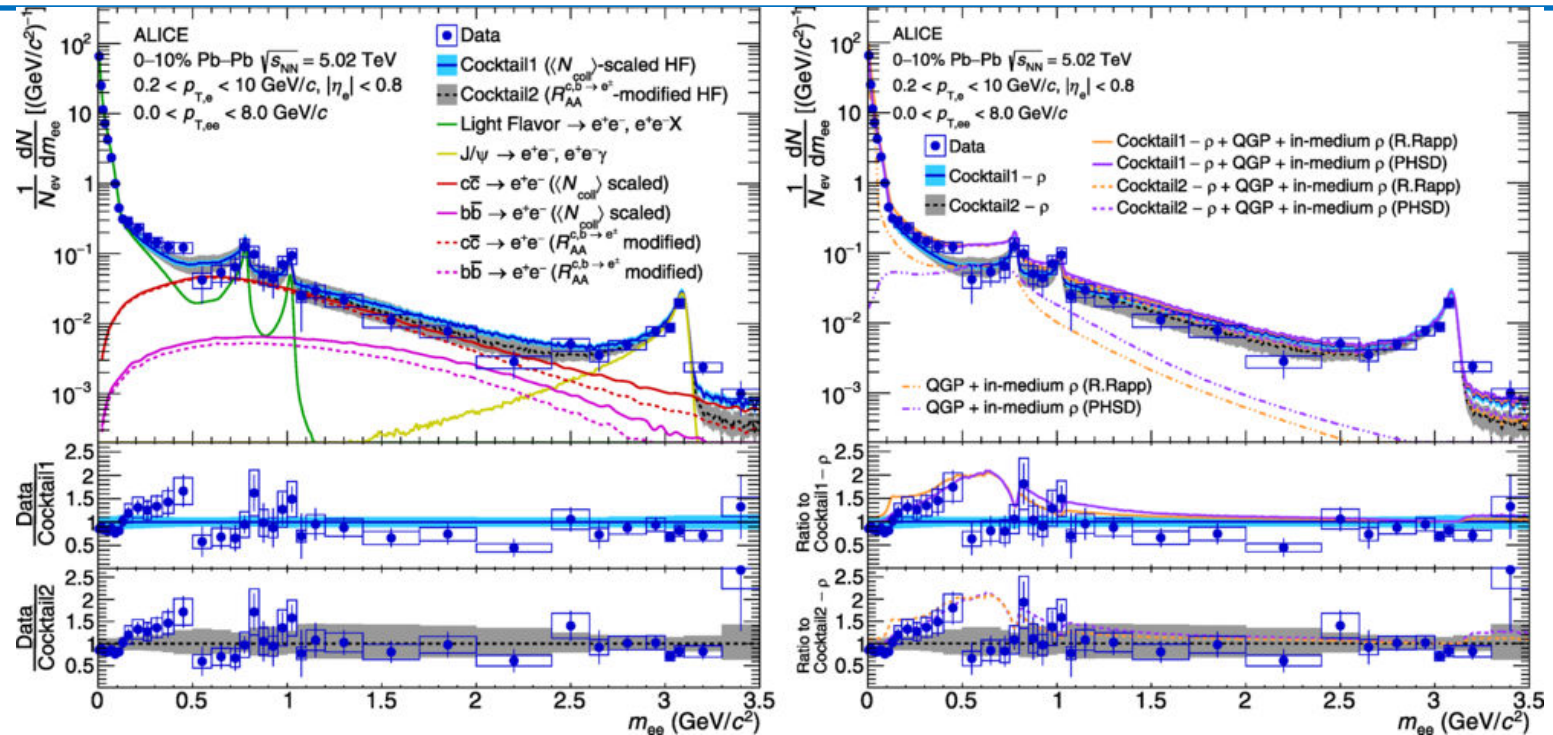
- $\rho(\omega, \vec{p}) = \int d^4x e^{i(\omega t - \vec{p} \cdot \vec{x})} \langle [J(x), J(0)] \rangle$
 - In euclidean time on lattice
- Connection to experiment
 - Low mass dileptons
 - Thermal photons
 - Quarkonia suppression
 - Heavy flavour flow

Dileptons and thermal photons

- pp
 - Vacuum baseline
- Pb-Pb
 - Chiral symmetry restoration
 - rho broadening
 - QGP temperature
- At LHC huge heavy flavour bckg



Dielectrons and thermal photons

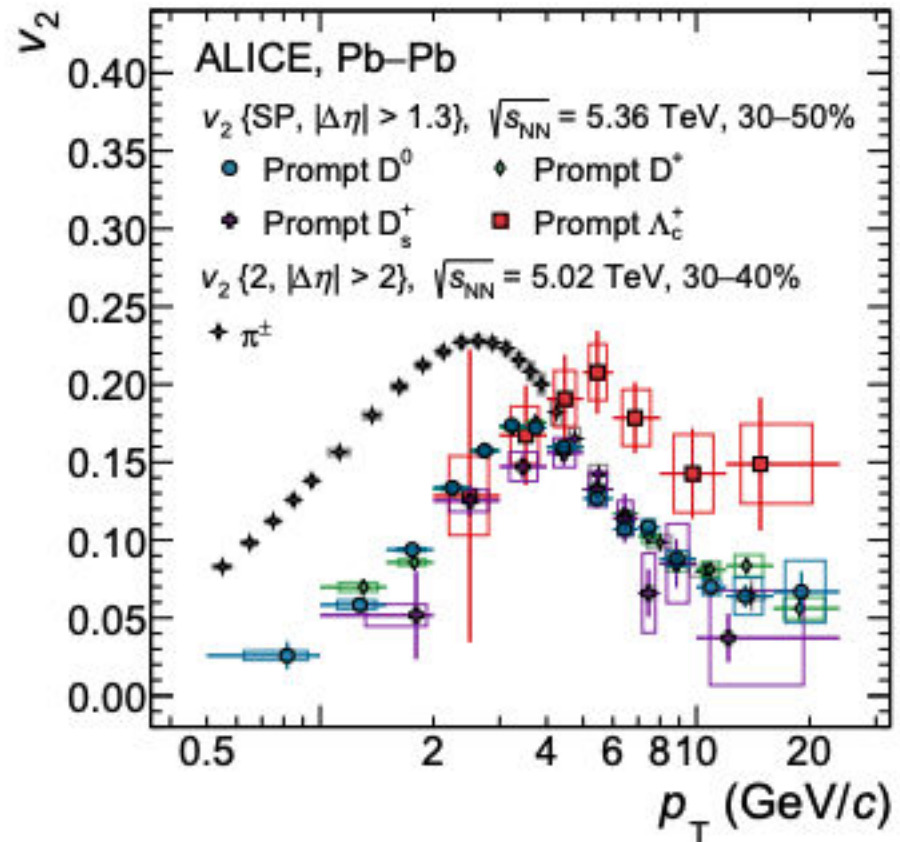


- [ALICE Run2](#) dielectron invariant mass spectrum
- Compared to
 - cocktail of expected hadronic decay contributions (left)
 - Prediction of thermal radiation from medium (right) including lattice rho broadening
- ALICE Run3 – 100 times more data – models more constrained

Heavy flavour flow - transport

- Run3 ALICE results
- <https://arxiv.org/pdf/2603.18966>

- D^0 and D^+ mesons are compatible within uncertainties in the full p_T interval of the measurement
- deviation of 2.6σ in $1 < p_T < 5$ GeV/c is observed between D^0 and D_s^+ mesons
- The prompt Λ_c^+ -baryon v_2 exceeds that of D mesons for $4 < p_T < 12$ GeV/c with a significance of 3.7σ .
- The baryon–meson splitting in the charm sector indicates a partonic origin of the flow.



Heavy flavour flow - transport

- Run3 ALICE results
- <https://arxiv.org/pdf/2603.18966>

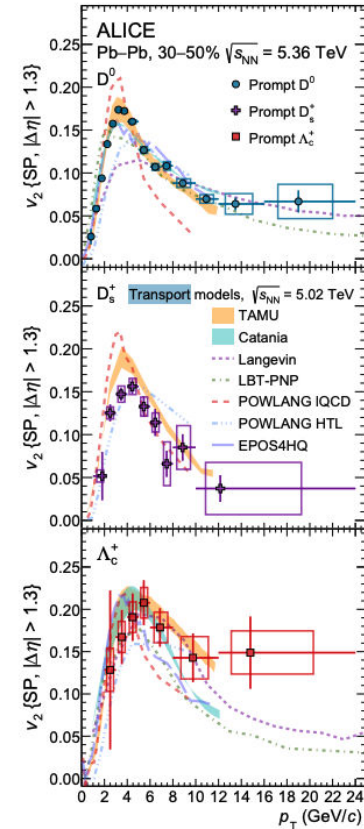
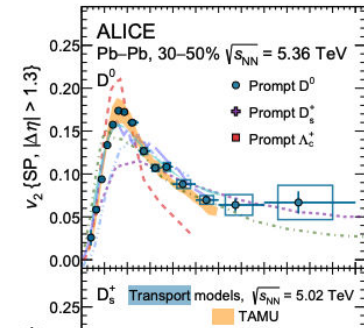


Figure 2: Elliptic flow of prompt D^0 (top), D_s^+ (middle), and Λ_c^+ (bottom) hadrons at midrapidity ($|y| < 0.8$) in Pb–Pb collisions at $\sqrt{s_{NN}} = 5.36$ TeV in the 30–50% centrality class as a function of p_T . The data are compared with predictions from TAMU [78], Catania [43, 79], POWLANG [80, 81], LBT-PNP [44, 75], EPOS4HQ [76], and Langevin [82, 83] transport models at $\sqrt{s_{NN}} = 5.02$ TeV.

Heavy flavour flow - transport

- Run3 ALICE results
- <https://arxiv.org/pdf/2603.18966>



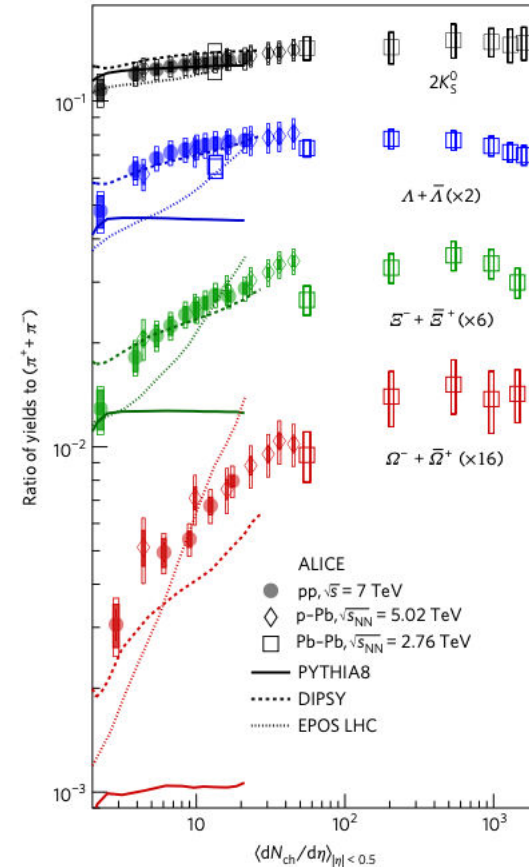
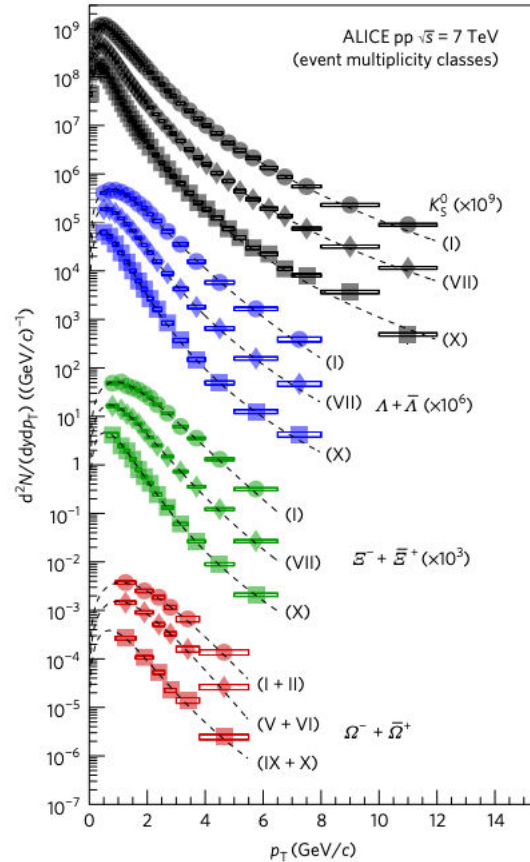
Model	Transport origin	Lattice QCD usage	Nature
Catania	quasi-particle	indirect	semi-phenomenological
EPOS4HQ	fitted	weak	data-driven
Langevin	flexible	often direct	framework
LBT-PNP	pQCD	minimal	perturbative
POWLANG-HTL	HTL pQCD	none	perturbative
POWLANG-IQCD	lattice input	direct	lattice-driven
TAMU	T-matrix	strong indirect	non-perturbative

Collectivity in small systems like pp and pA ?

Strangeness from pp to PbPb

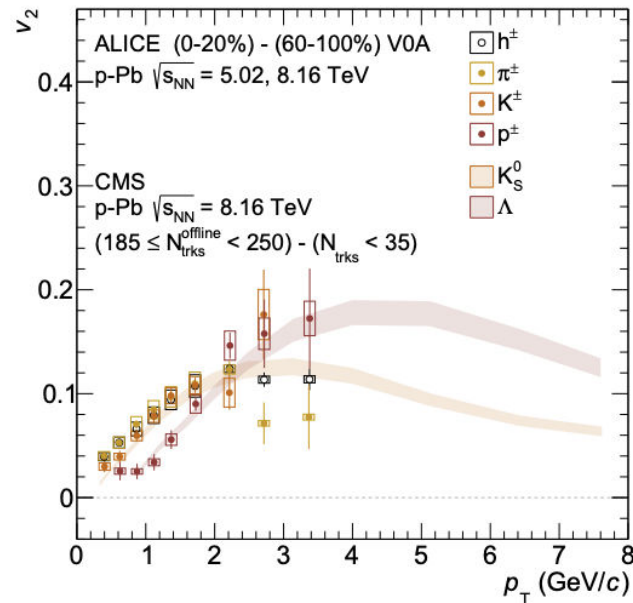
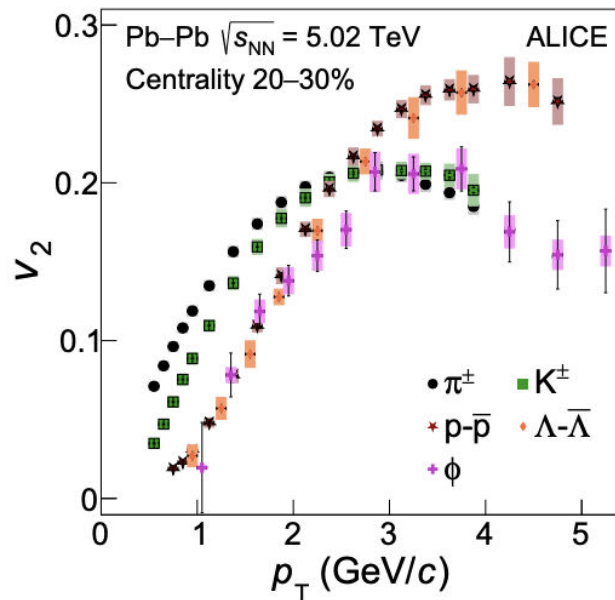
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NATURE PHYSICS DOI: 10.1038/NPHY



pPb flow

- ALICE: <https://arxiv.org/abs/2211.04384>



pp and Pb flow

■ pp and pPb flows

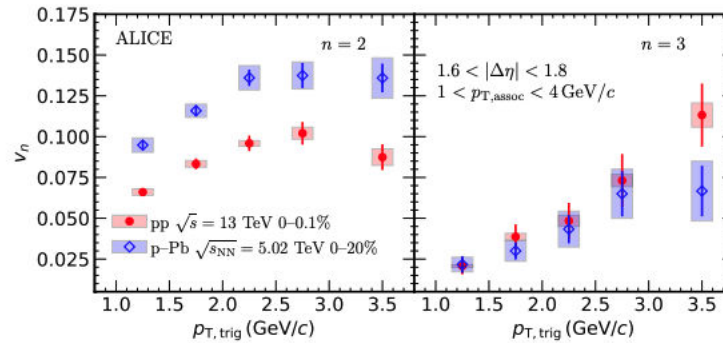
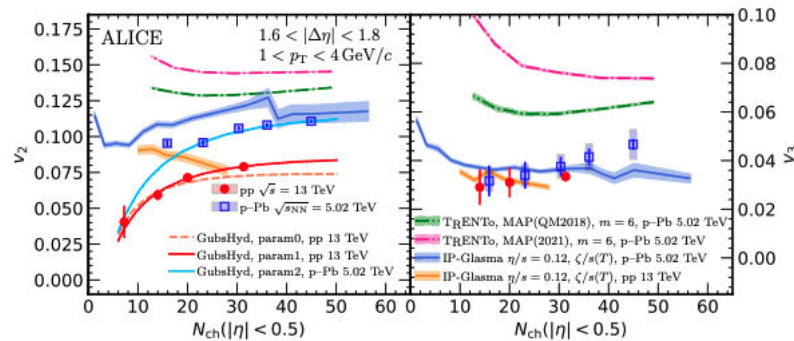


Figure 4: The magnitude of v_2 (left) and v_3 (right) as a function of p_T for the 0–0.1% multiplicity interval in pp collisions at $\sqrt{s} = 13$ TeV and 0–20% in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV. The boxes around the data points represent the estimated systematic uncertainty and the error bars correspond to statistical errors.

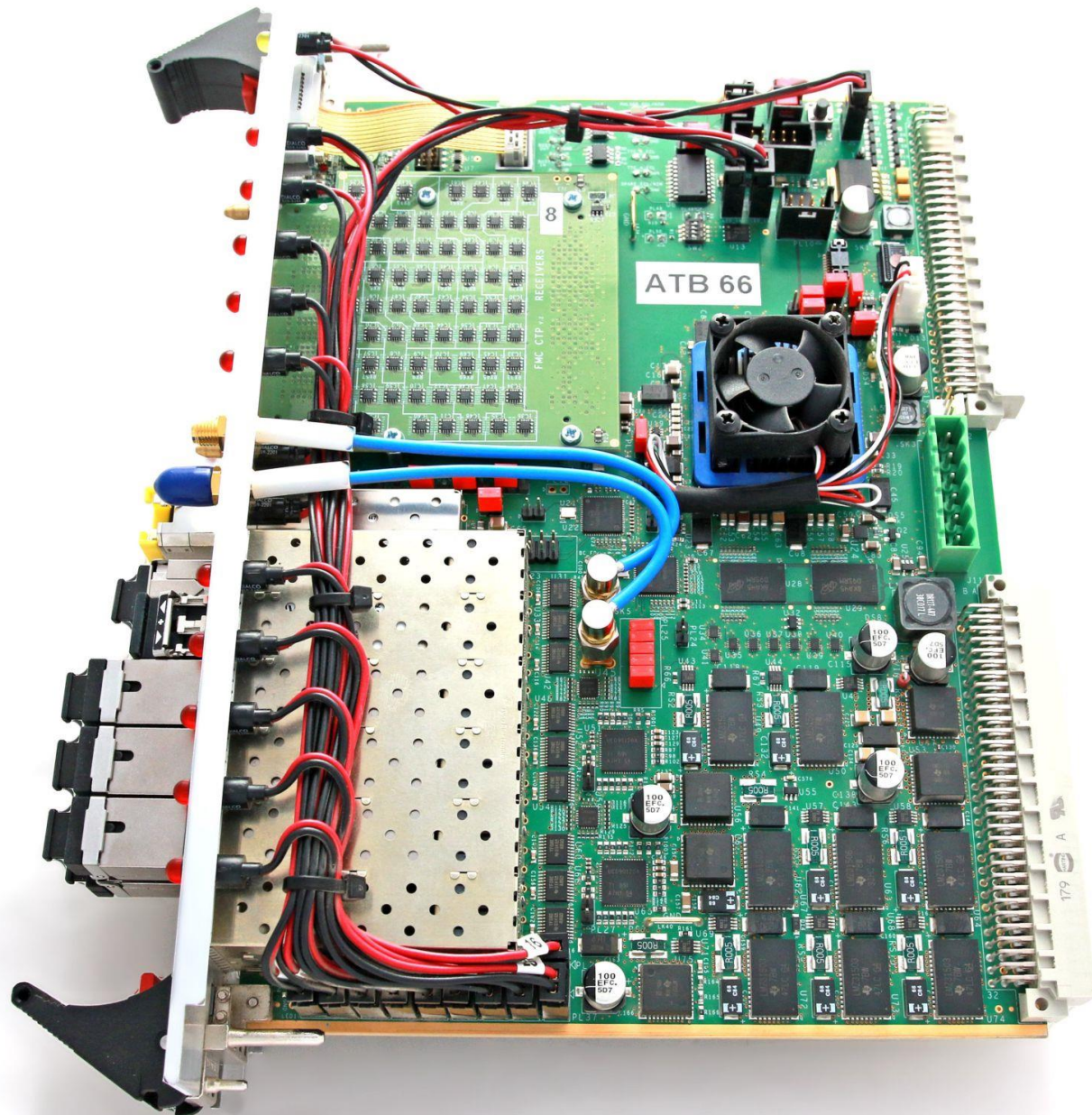


Summary

- In Pb-Pb collisions at the LHC, a sizeable fireball with initial temperature $T > 300 \text{ MeV}$ is created
- Hydrodynamics describes the expansion of the fireball $\Rightarrow$ system behaves like an almost ideal liquid
 - degrees of freedom are quarks and gluons
- Lattice QCD $\Leftrightarrow$ HI observable connection nontrivial
 - Ideal thermodynamics versus expanding HI system
- Some QGP signatures observed also in high multiplicity p-Pb and pp systems
 - What is the smallest size for QGP fluid ?



■ Backup



Blast wave

Hydro picture:

m_T spectra sensitive to the transverse flow

Blast wave description of the spectra:

$$\frac{d^2 N_j}{m_T dy dm_T} = \int_0^{R_G} A_j m_T \cdot K_1 \left(\frac{m_t \cosh \rho}{T} \right) \cdot I_0 \left(\frac{p_t \sinh \rho}{T} \right) r dr$$

$$\rho(r) = \tanh^{-1} \beta_{\perp}(r)$$

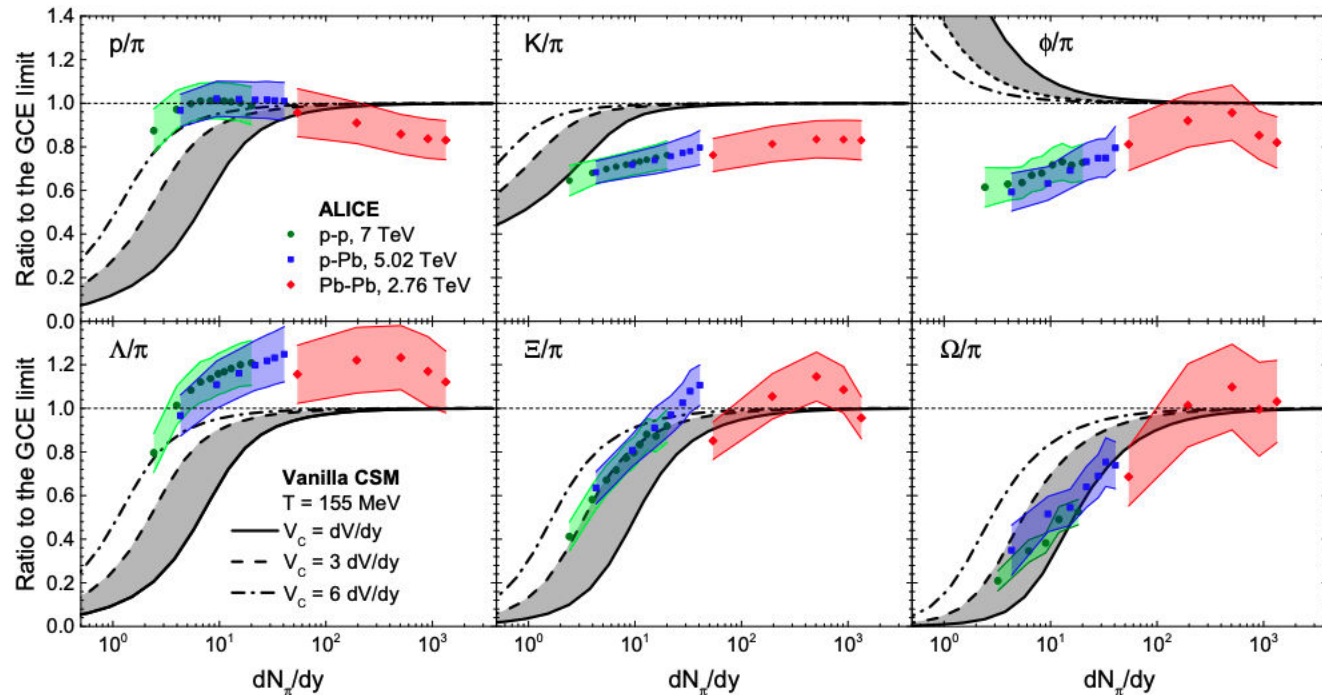
$$\beta_{\perp}(r) = \beta_s \left[\frac{r}{R_G} \right]^n \quad r \leq R_G$$

Uniform particle density

$$\langle \beta_{\perp} \rangle = \frac{2}{2+n} \beta_s$$

Ref: E Schnedermann, J Sollfrank and U Heinz, **Phys. Rev. C48** (1993) 2462

Thermal model - CE



- ALICE data
- Canonical suppression
- Model: <https://arxiv.org/pdf/1906.03145>
- Reasonable description

$v_n \{2k\}$ for Gauss Bessel

$$v_n \{2\}^2 \equiv \langle v_n^2 \rangle,$$

$$v_n \{4\}^4 \equiv -\langle v_n^4 \rangle + 2\langle v_n^2 \rangle^2,$$

$$v_n \{6\}^6 \equiv \left(\langle v_n^6 \rangle - 9\langle v_n^4 \rangle \langle v_n^2 \rangle + 12\langle v_n^2 \rangle^3 \right) / 4,$$

$$v_n \{8\}^8 \equiv - \left(\langle v_n^8 \rangle - 16\langle v_n^6 \rangle \langle v_n^2 \rangle - 18\langle v_n^4 \rangle^2 + 144\langle v_n^4 \rangle \langle v_n^2 \rangle^2 - 144\langle v_n^2 \rangle^4 \right) / 33$$

:

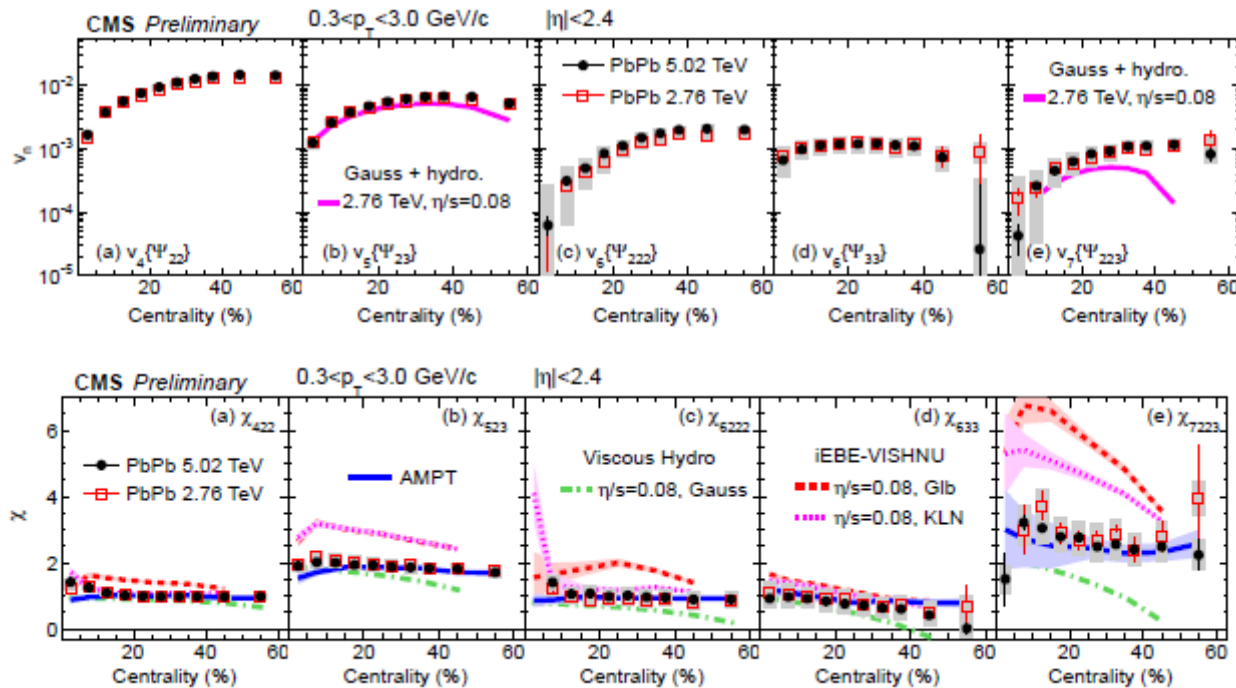
$$p(v_n) = \frac{v_n}{\delta_{v_n}^2} \exp \left[-\frac{(v_n)^2 + (v_n^{RP})^2}{2\delta_{v_n}^2} \right] I_0 \left(\frac{v_n v_n^{RP}}{\delta_{v_n}^2} \right),$$

$$v_n \{2k\} = \begin{cases} \sqrt{(v_n^{RP})^2 + 2\delta_{v_n}^2} & k = 1 \\ v_n^{RP} & k > 1 \end{cases}$$

- Constant 2k harmonic -> consistent with Gauss-Bessel
- One can extract sigma

CMS Plane correlations

■ HIN-16-018



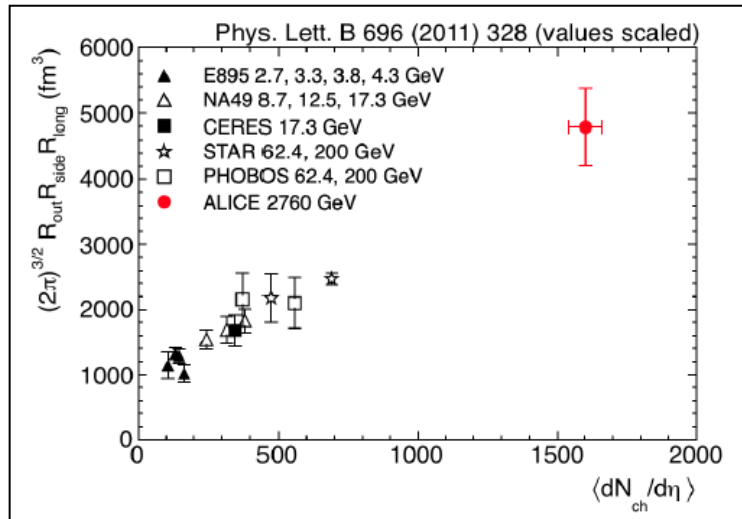
The data are compared with AMPT and hydrodynamic predictions with different shear viscosity to entropy density ratios and initial condition models. The predictions from AMPT are favored by the measurement. These results will provide constraints on the theoretical description of the medium close to the freeze-out temperature, which is poorly understood so far.

Geometry and flow

$$\varepsilon_2 = \langle e(x, y) \cos(2(\varphi - \psi_2)) \rangle^{v_2 \sim \epsilon_2}$$

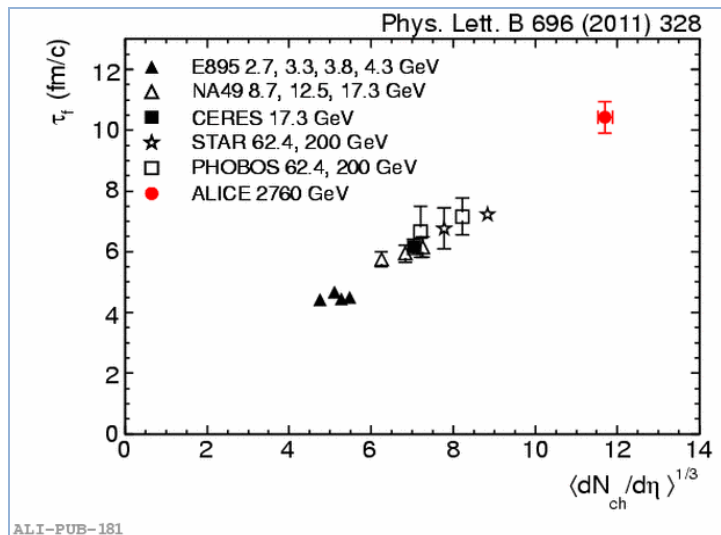
$e(x, y)$ – distribution of energy density of initial state

HBT radii – *System Size*

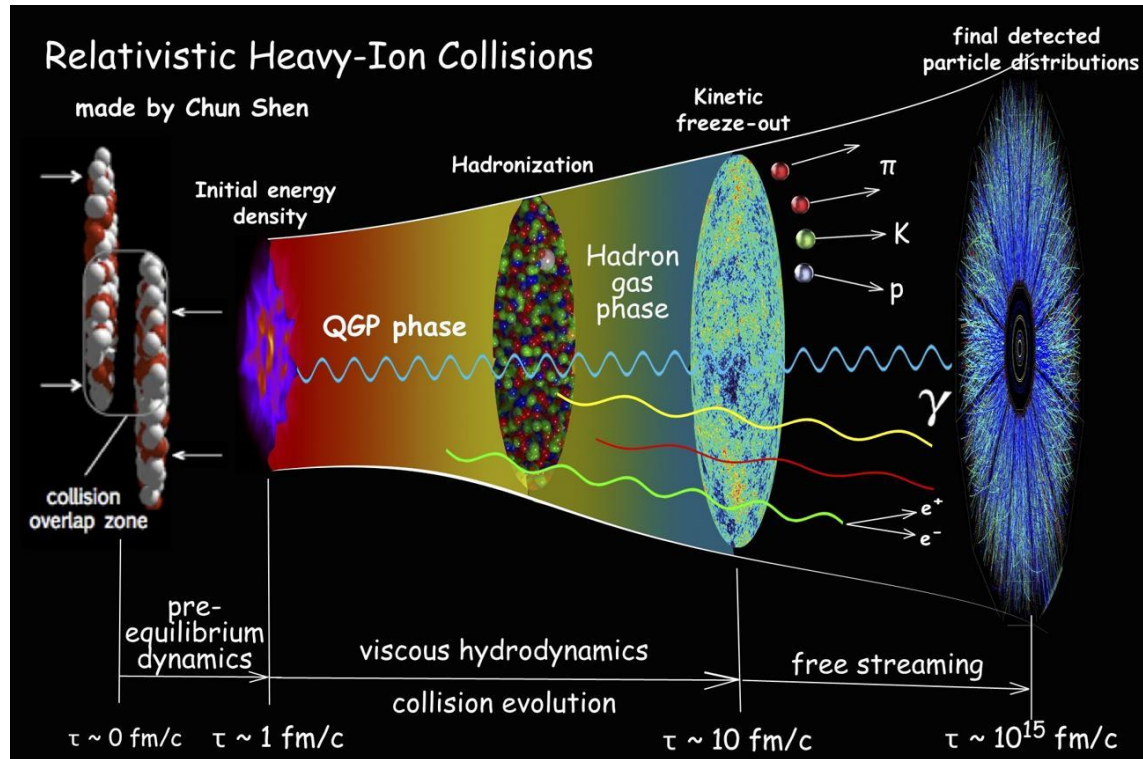


QM Interference of identical particles
(Hanbury-Brown & Twiss interferometry)
allows to measure source size and duration
of emission.

Fireball volume at freeze-out:
 $V \approx 5000 \text{ fm}^3 \approx 4 \times \text{Pb volume}$



Lifetime $\sim 10 \times \text{QCD scale } (1/\Lambda_{\text{QCD}})$



Beyond v_n

- Optimise sensitivity of flow related variable to disentangle medium properties (η/s) and initial conditions
 - Direct event by event measurement of $p(v_n)$
 - properties of probability distribution function $p(v_2)$
 - $v_2\{2\}^2 = \langle v_n^2 \rangle$ but if pdf $p(v_2) \neq \delta(v_2)$ then $\langle v_n^2 \rangle \neq \langle v_n \rangle^2$
 - Correlations
 - Fourier coefficients may be correlated due to physics
 - Correlate flows v_n , event planes ψ_n and their combinations in different phase space
 - $c(X(x), n; Y(y), m) = \langle X_n(x) Y_m(y) \rangle - \langle X_n(x) \rangle \langle Y_m(y) \rangle$

Elliptic Flow fluctuations

- Technique: Unfolding directly pdf of $p(v_2)$ (see ATLAS JHEP 11(2013) 183)
 - $v_2\{2\}, v_2\{4\}, v_2\{6\}, v_2\{8\}$ calculated using $p(v_2)$
- $p(\vec{v}_2)$ - usually assumed to be Gaussian
- $v_2\{4\} \sim v_2\{6\} \sim v_2\{8\}$
 - splitting characterise departure from Gauss of $p(v_2)$

