New trends in thermal phases of QCD Bratislava, 09 May 2026

The QCD chiral transition and the role of (would-be) Goldstone bosons

Owe Philipsen

- Chiral phase transition in massless limit Nf=2+1
- Chiral phase transition for many flavours and onset of conformal window
- Thermoparticles and Goldstone bosons

CRC-TR 211 Strong-interaction matter under extreme conditions

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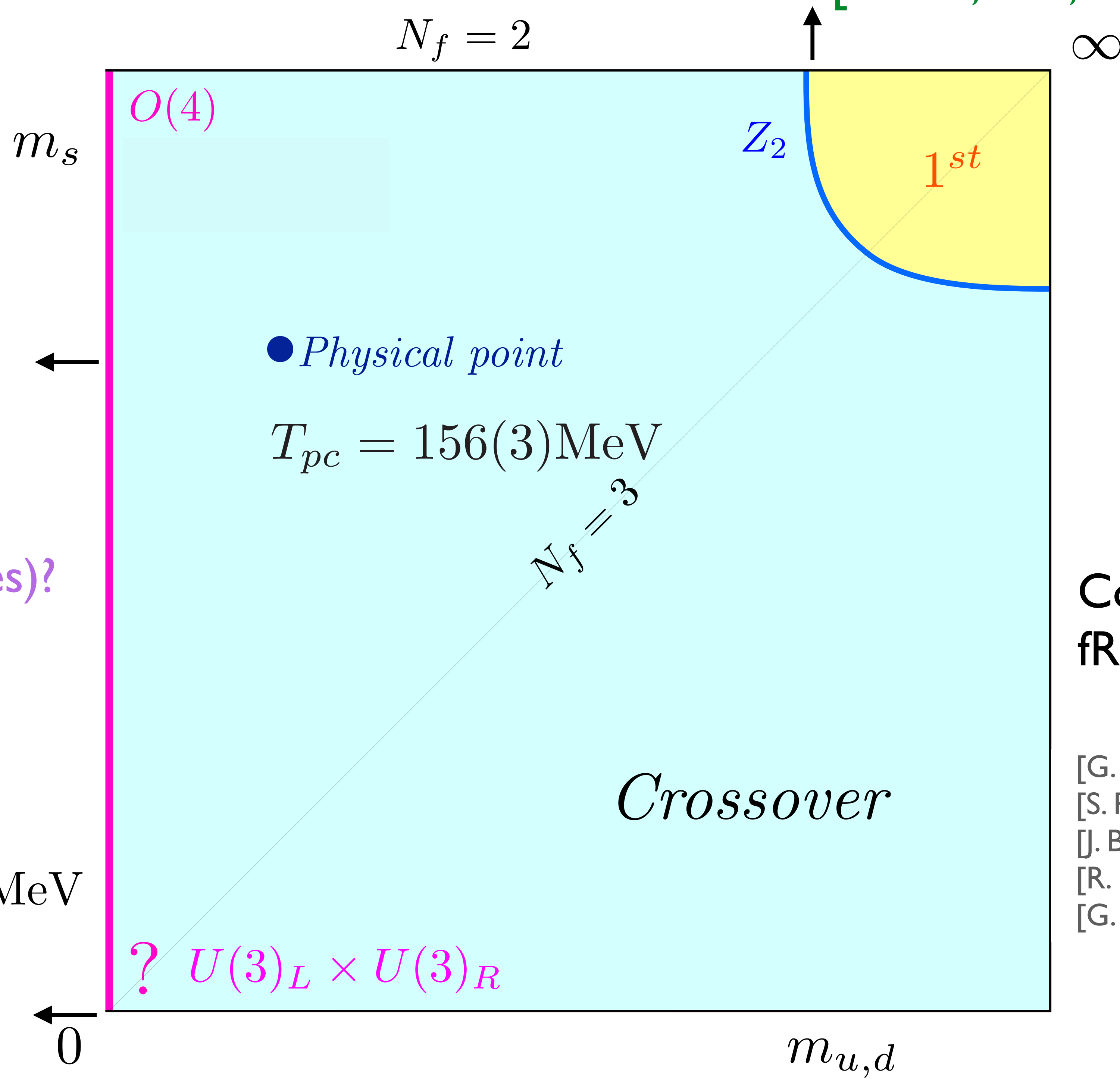
The Columbia plot in the continuum

[Cuteri, O.P., Sciarra JHEP 21]

$m_\pi \approx 3.5 - 4$ MeV

[Kaiser, O.P., PoS LAT22]



[HotQCD PRL 19]

$T_c^0 = 135(8)$MeV

Universality class(es)?

[HotQCD PRD 22]

HISQ, crossover down to $m_\pi \approx 50$ MeV

$T_c^0 = 98(6)$MeV

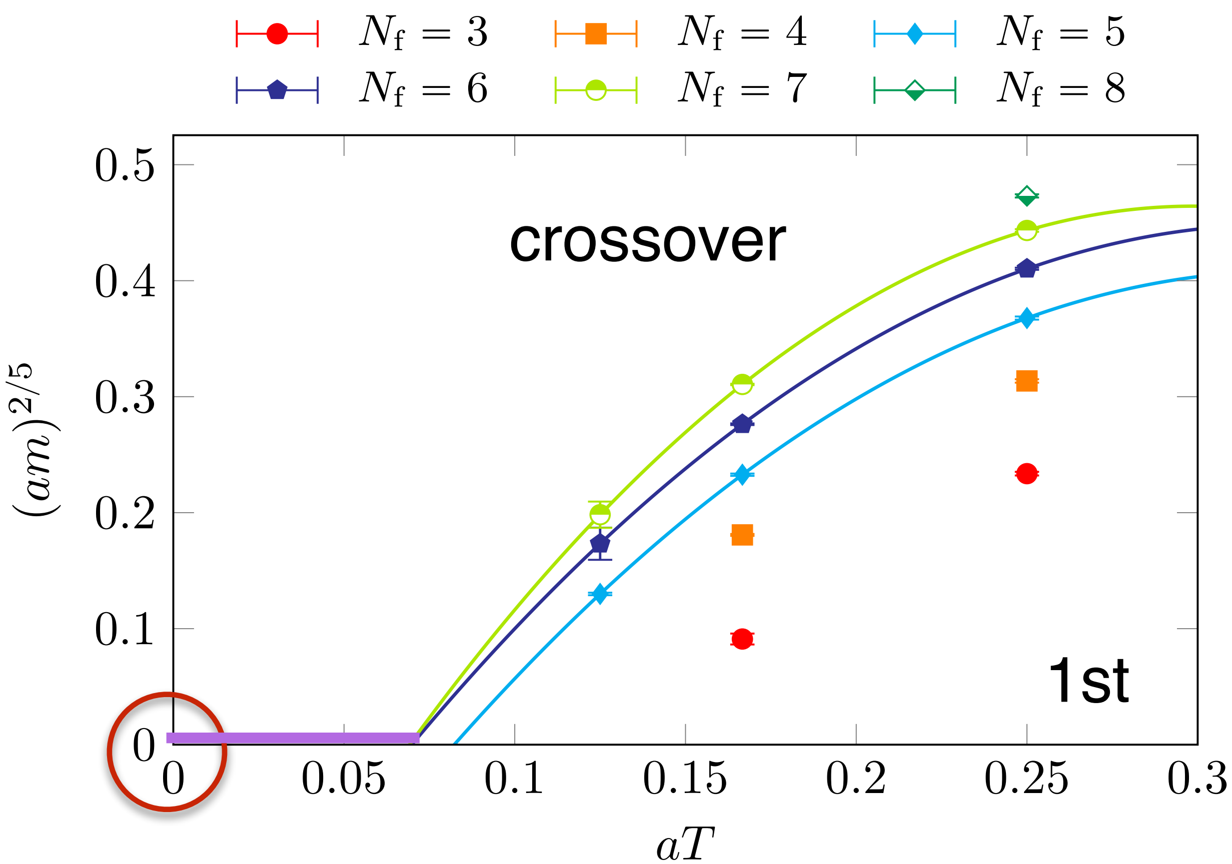
Consistent with:
fRG, DSE, bootstrap…

[G. Fejos, PRD 22]
[S. R. Kousvos, A. Stergiou SciPost 23]
[J. Bernhardt, C. S. Fischer PRD 23]
[R. D. Pisarski, F. Rennecke PRL 24]
[G. Fejos, T. Hatsuda PRD 24]

Crossover for DW fermions, Nf=3, $m_q \sim m_{u,d}^{\rm phys}$ [Zhang et al., PoS LAT22, 23]

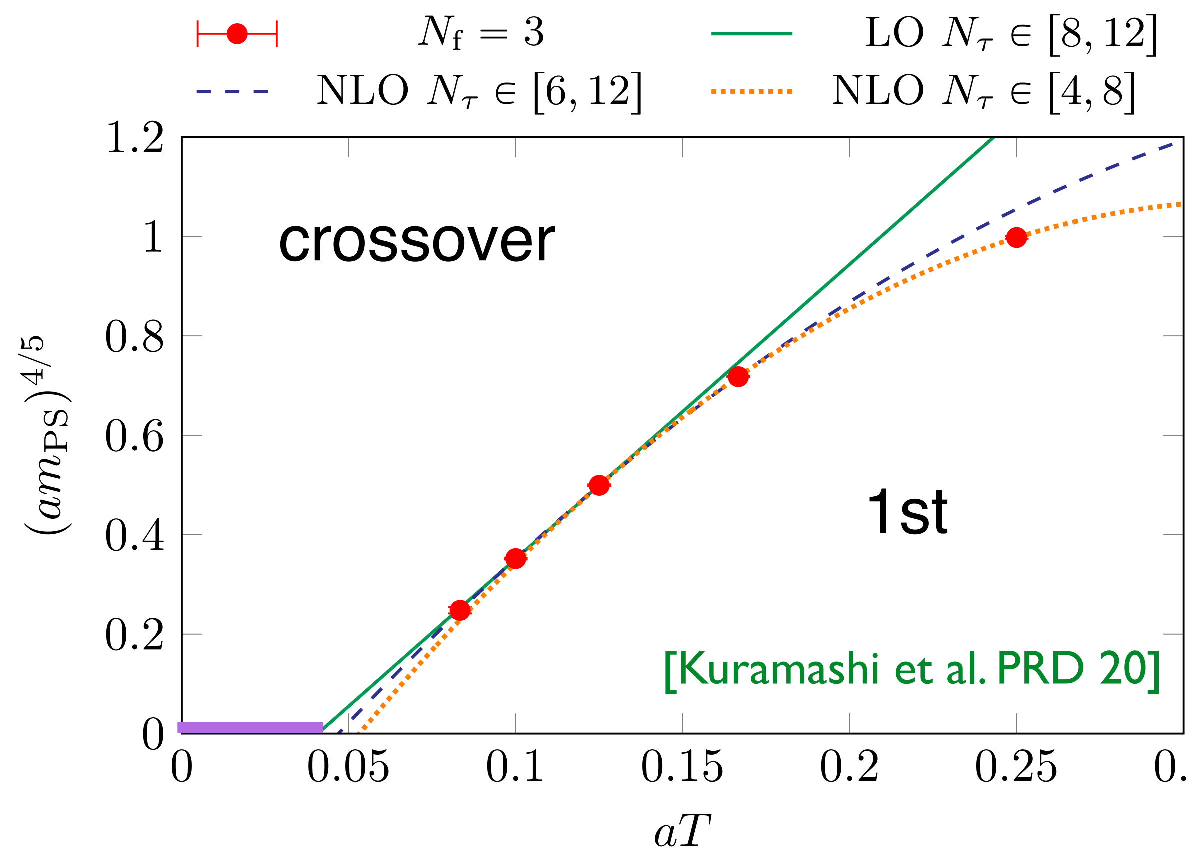
Endpoint of 1.O. transition in lattice parameter space

[Cuteri, O.P., Sciarra JHEP 21] Staggered



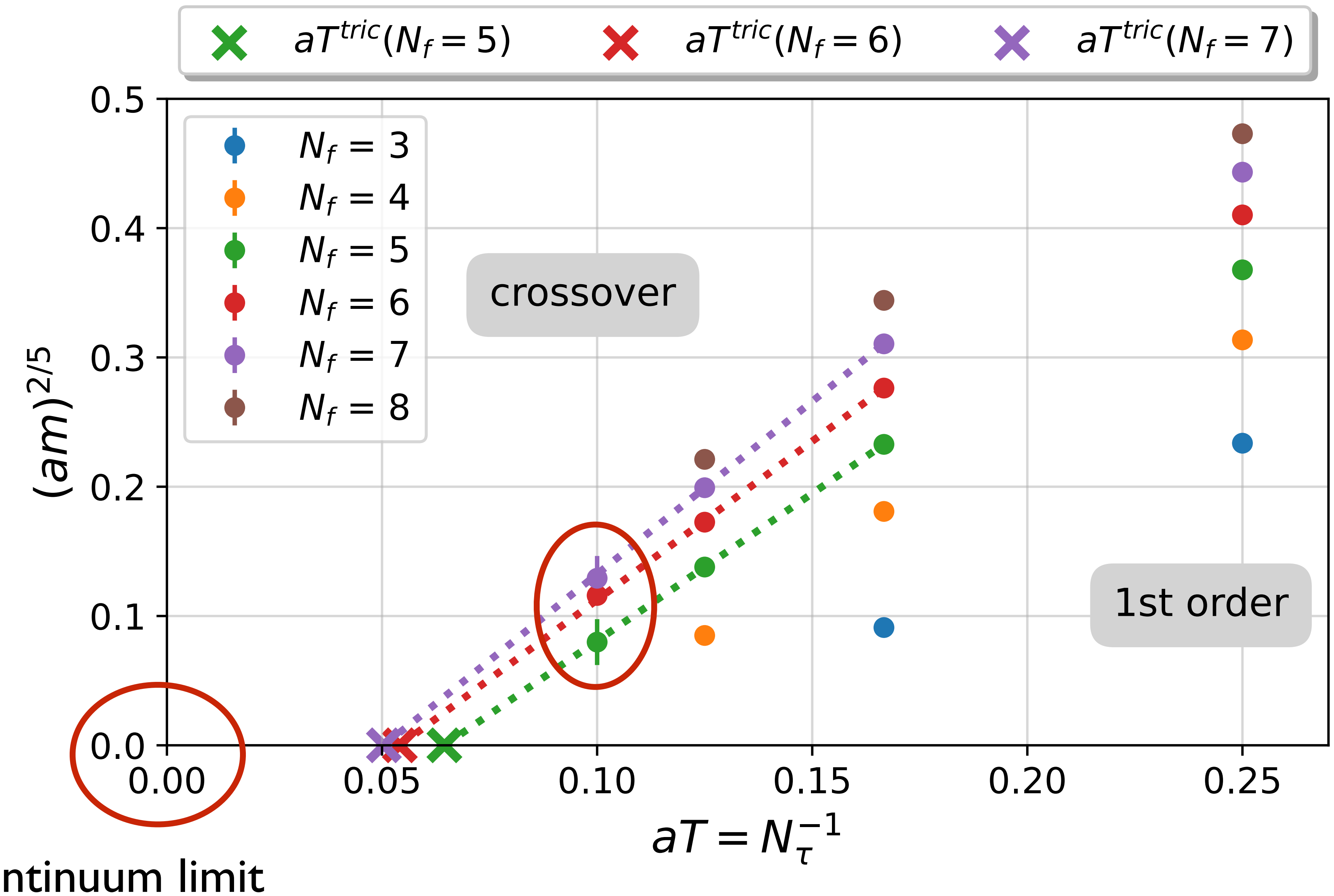
continuum limit

O(a) improved Wilson rescaled



- Tricritical scaling observed in lattice bare parameter space
- Allows extrapolation to lattice chiral limit, tricritical points $N_\tau^{\text{tric}}(N_f)$
- If tricritical point exists: region of 1st-order transitions not connected to continuum
- QCD chiral transition is second order for $N_f = 2 - 7$

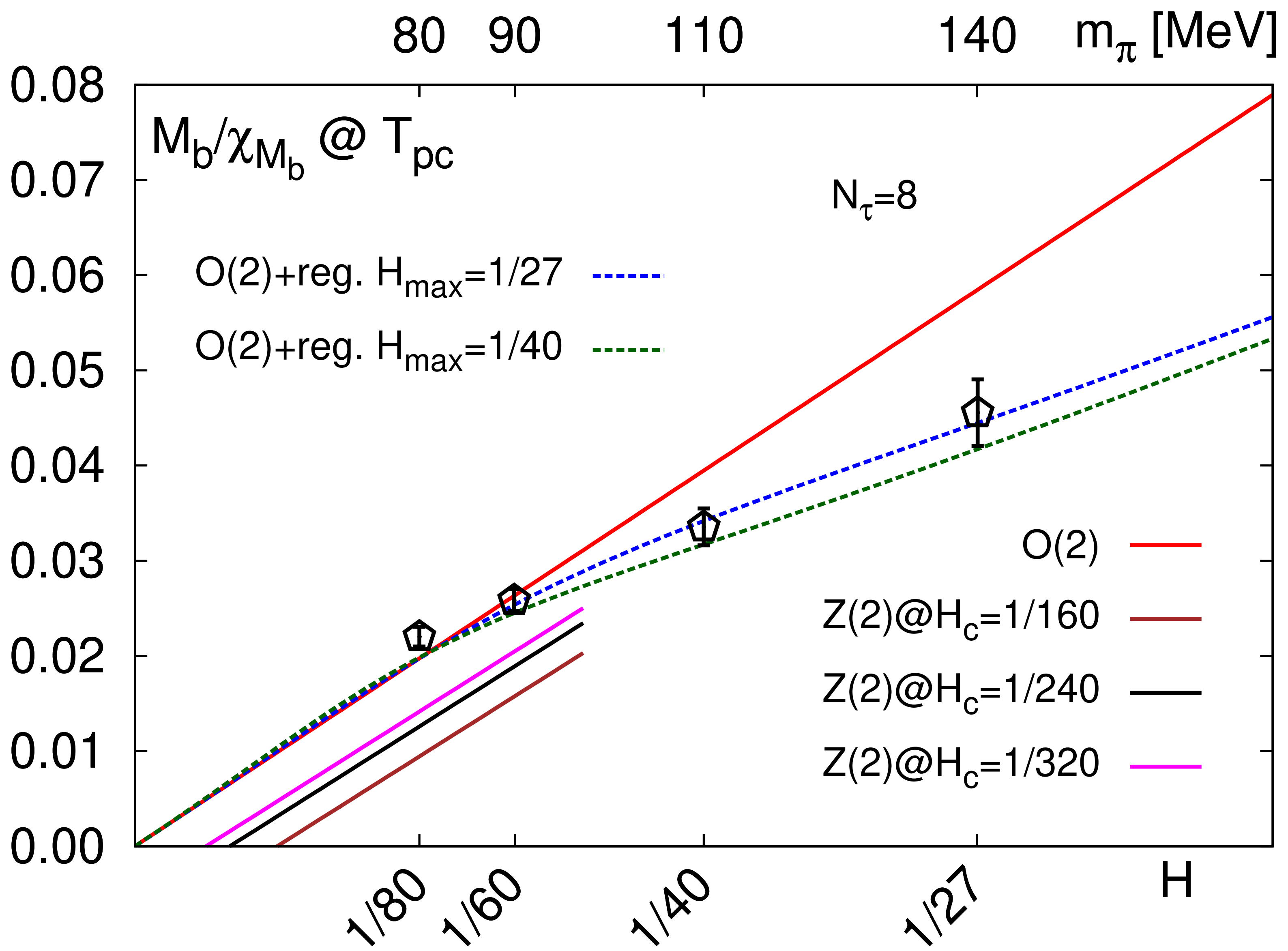
In progress: confirmation on finer lattices



New $N_\tau = 10$ results confirm tricritical scaling! [Frankfurt, preliminary]

Alternative approach

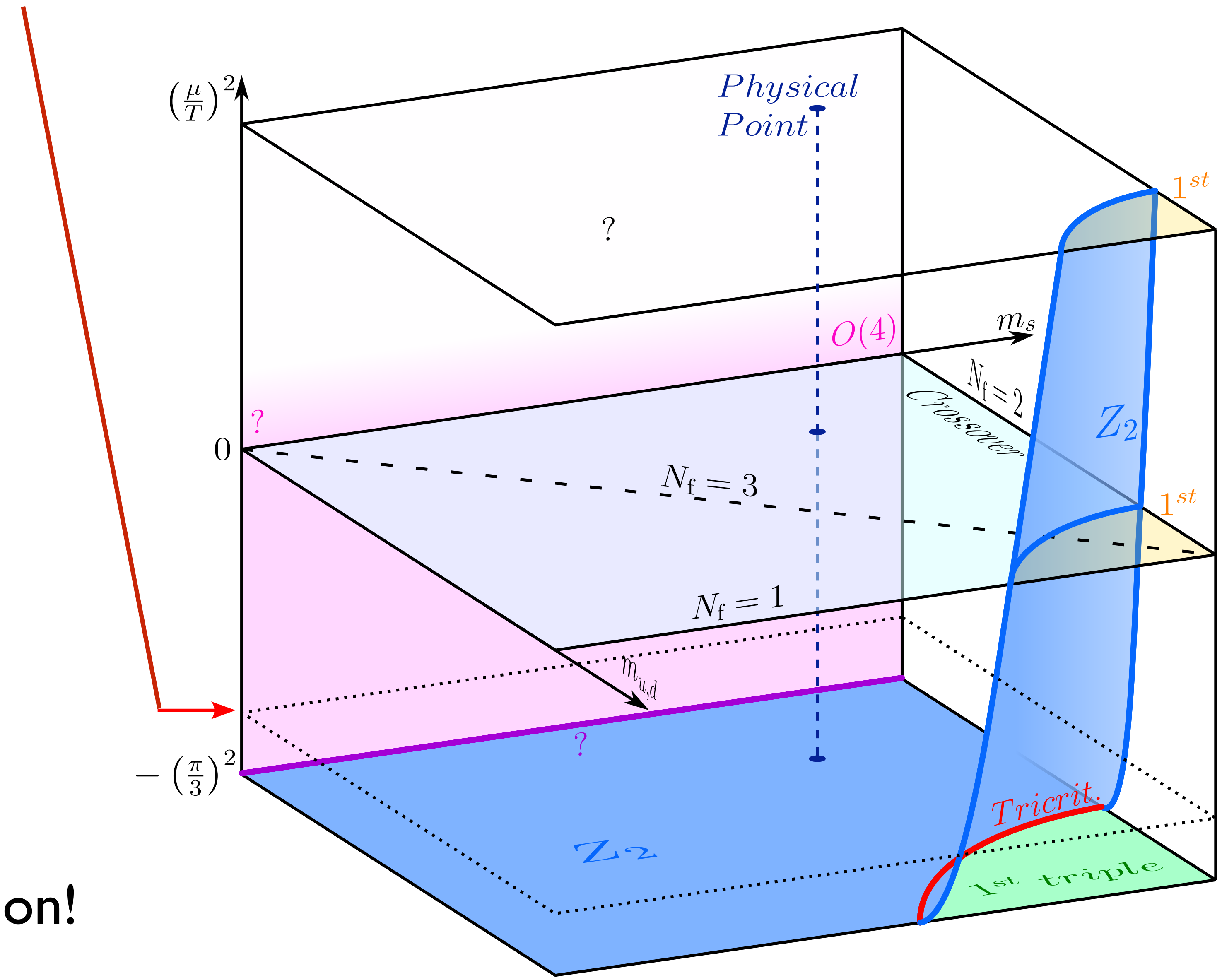
[HotQCD, PRD 22]



Imaginary chemical potential

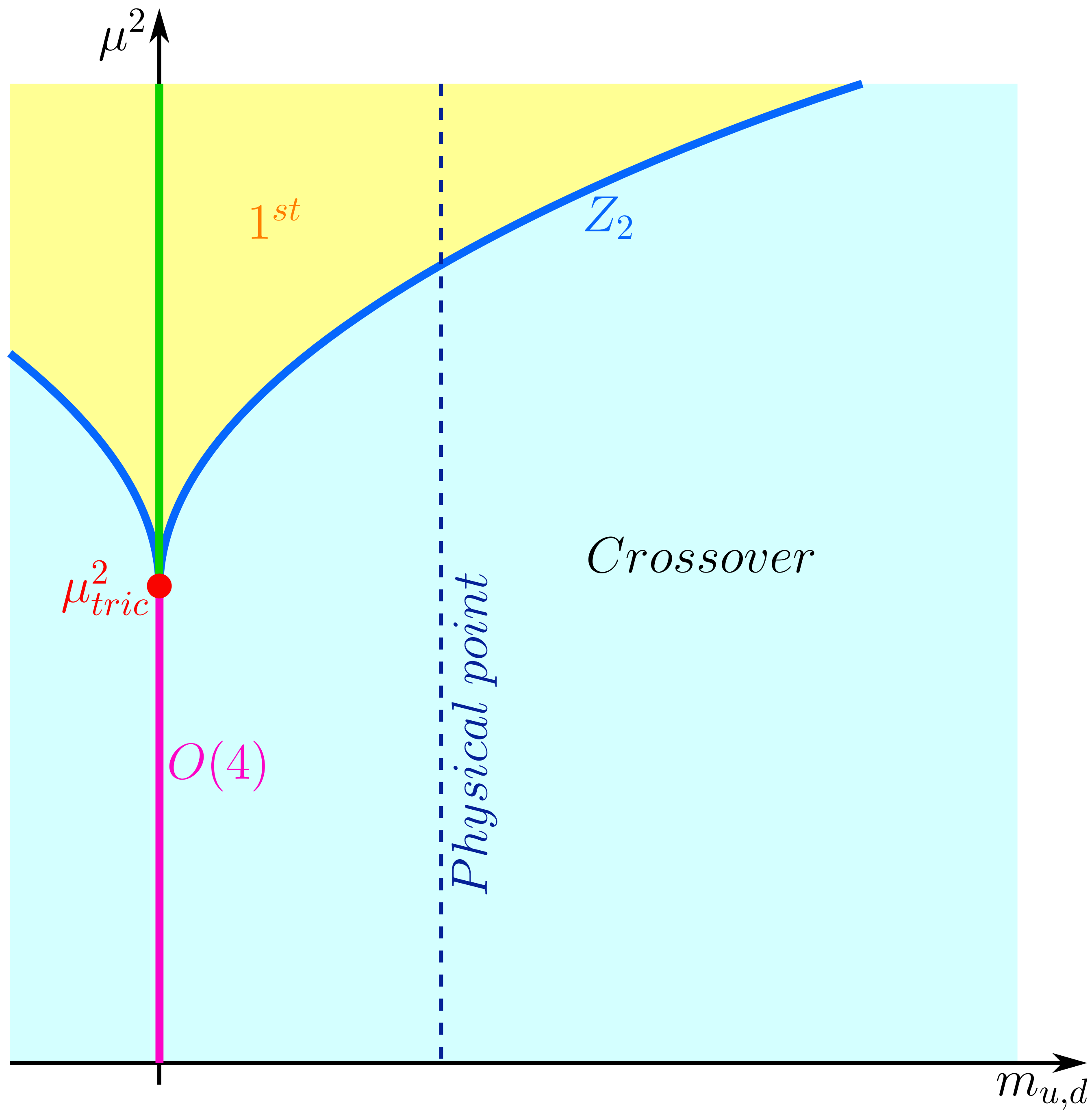
No sign problem! [D'Ambrosio, Fromm, Kaiser, O.P., EPJC 26]

Repeat study of Columbia plot with $\mu = i\,0.81\pi T/3$ Same situation as $\mu = 0$



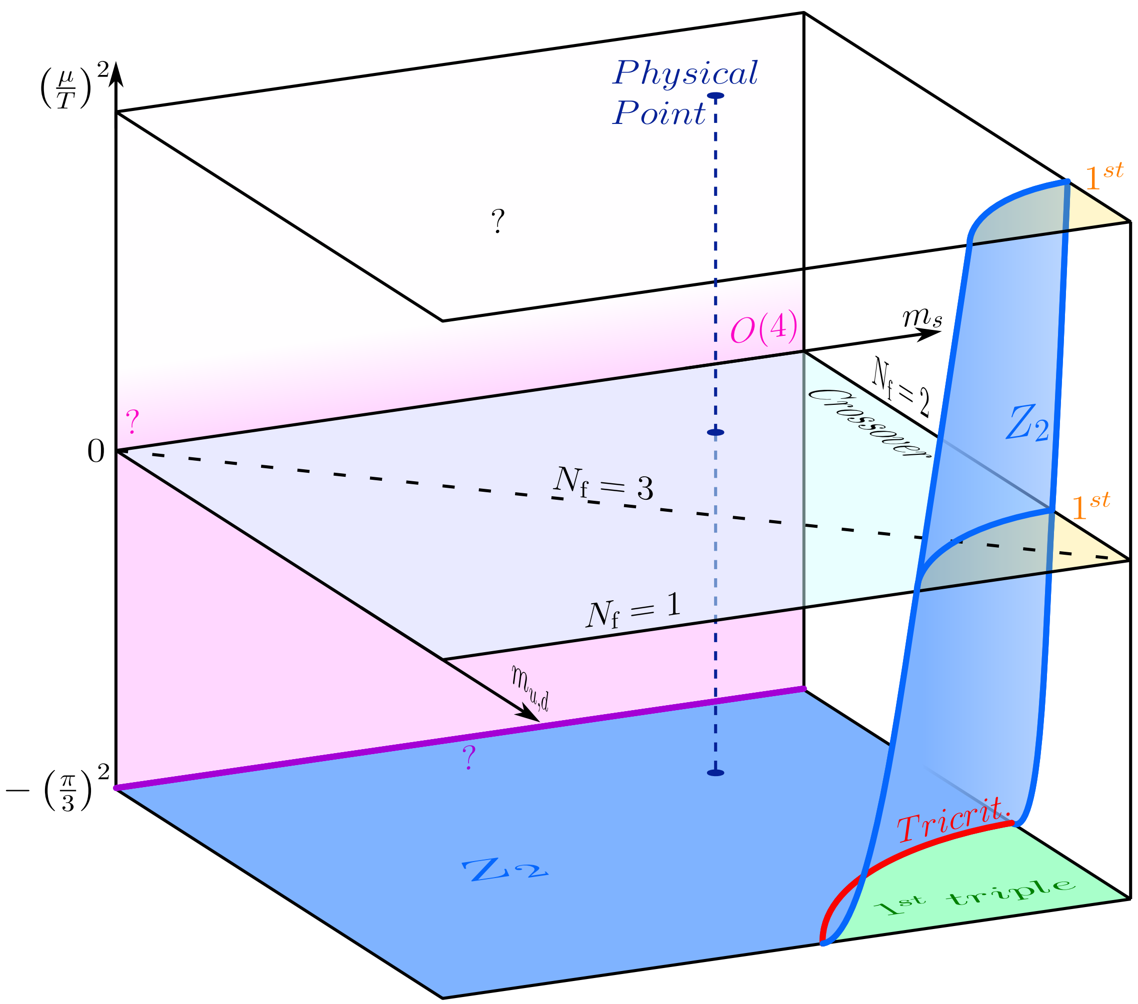
- [Bonati et al., PRD 19]
  stout-smeared staggered
  $\mu = i\pi T/3$
- [Bielefeld+Frankfurt, PRD 22]
  HISQ $\mu = i\pi T/3$
- No sign of 1st-order phase transition!
- Consistent with DSE approach [Bernhardt, Fischer, PRD 23, arXiv 25]

Crit. Endpoint: non-analytic chiral surface/line



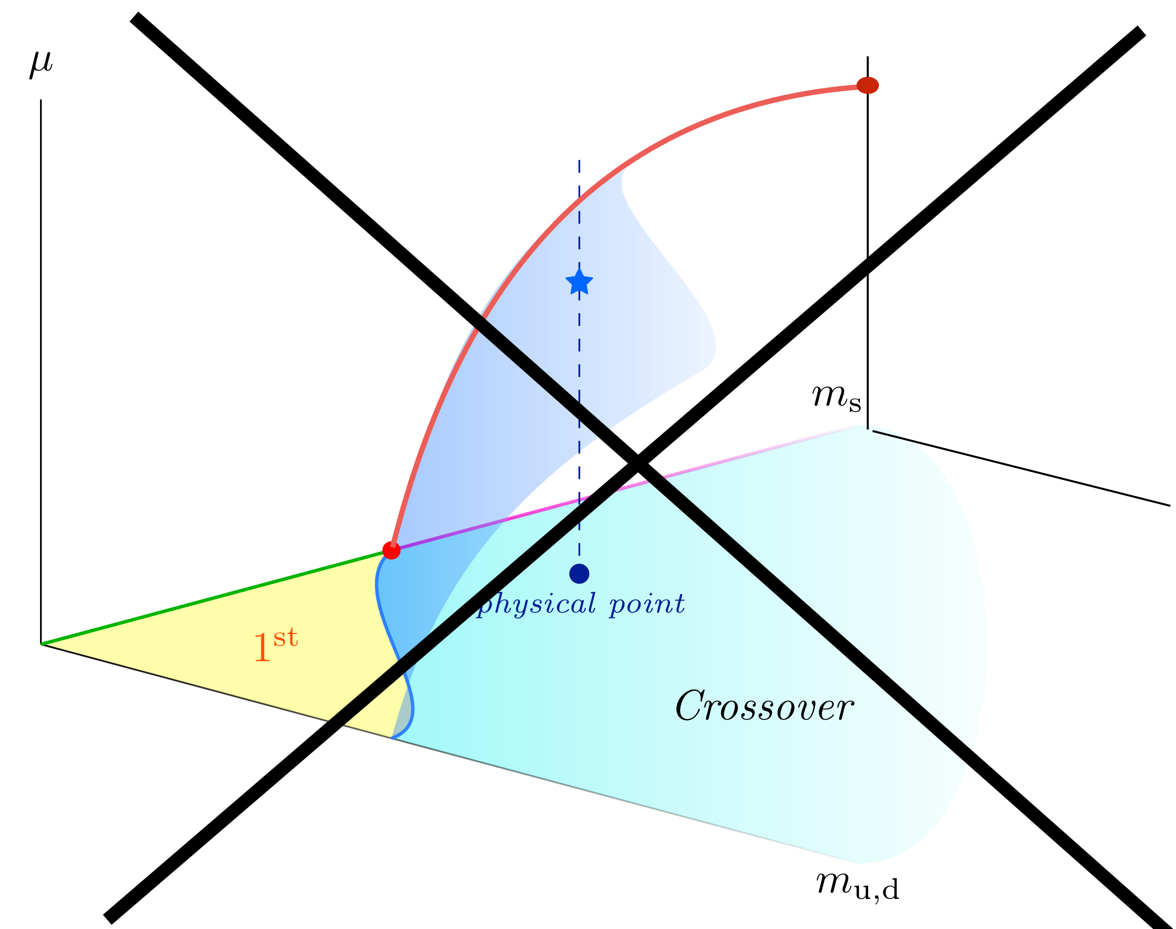
Columbia plot with chemical potential

Critical point not ruled out, but requires additional critical surface



Class of low energy models now ruled out!

[Stephanov, Rajagopal, Shuryak PRL 98]:

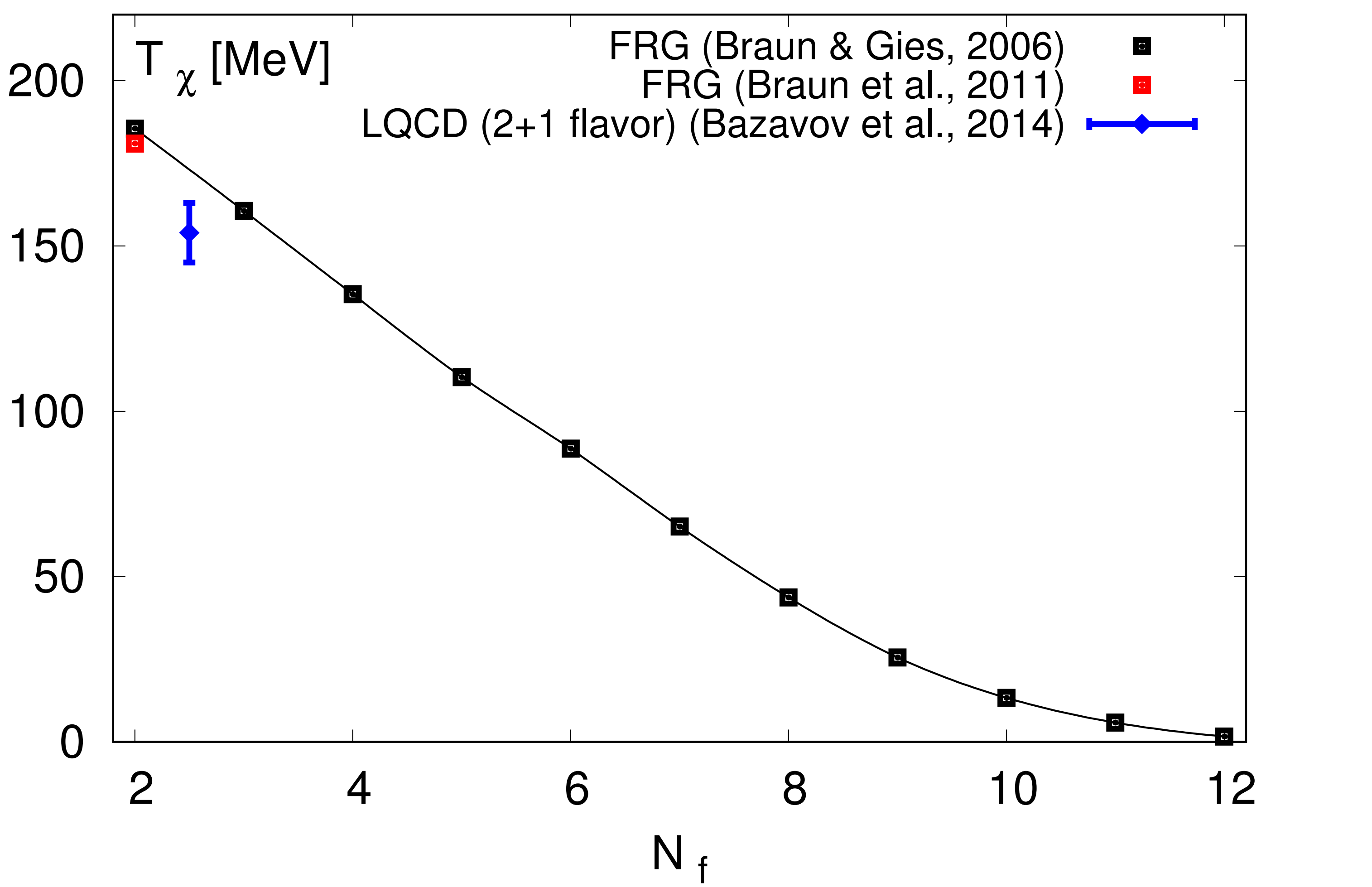


[Fig. modified from Sciarra, PhD thesis 16]

$\nexists$ Tuning of parameters for $N_f = 2+1$ theory with critical point at $\mu = 0$!

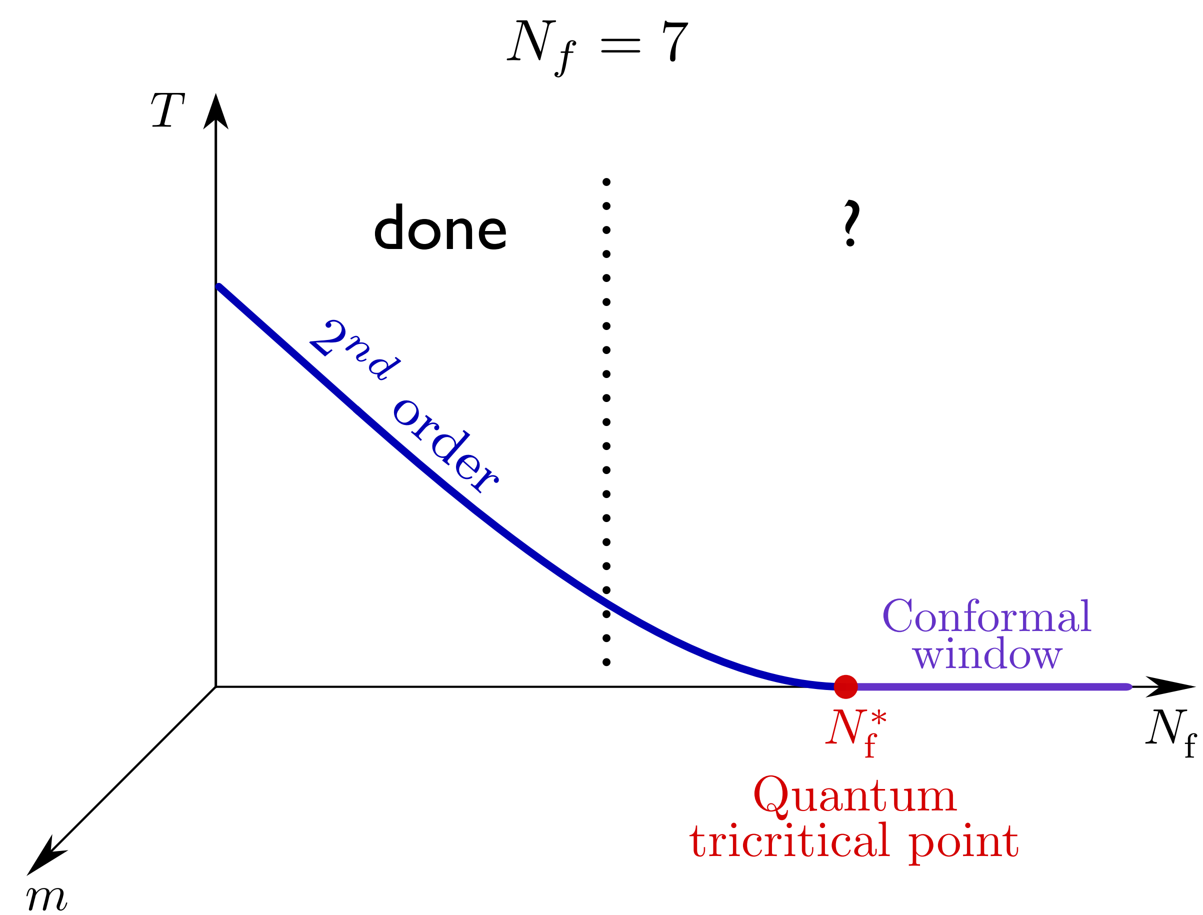
The chiral phase transition as a function of N_f

Temperature dependence:



For lattice, see [Miura, Lombardo, NPB 13]

Order of the transition:



[Cuteri, O.P., Sciarra, JHEP 21]

The chiral phase transition in the massless limit is likely second-order for all N_f

Towards the conformal window, $N_f > 7$

What is the value of N_f^* ?

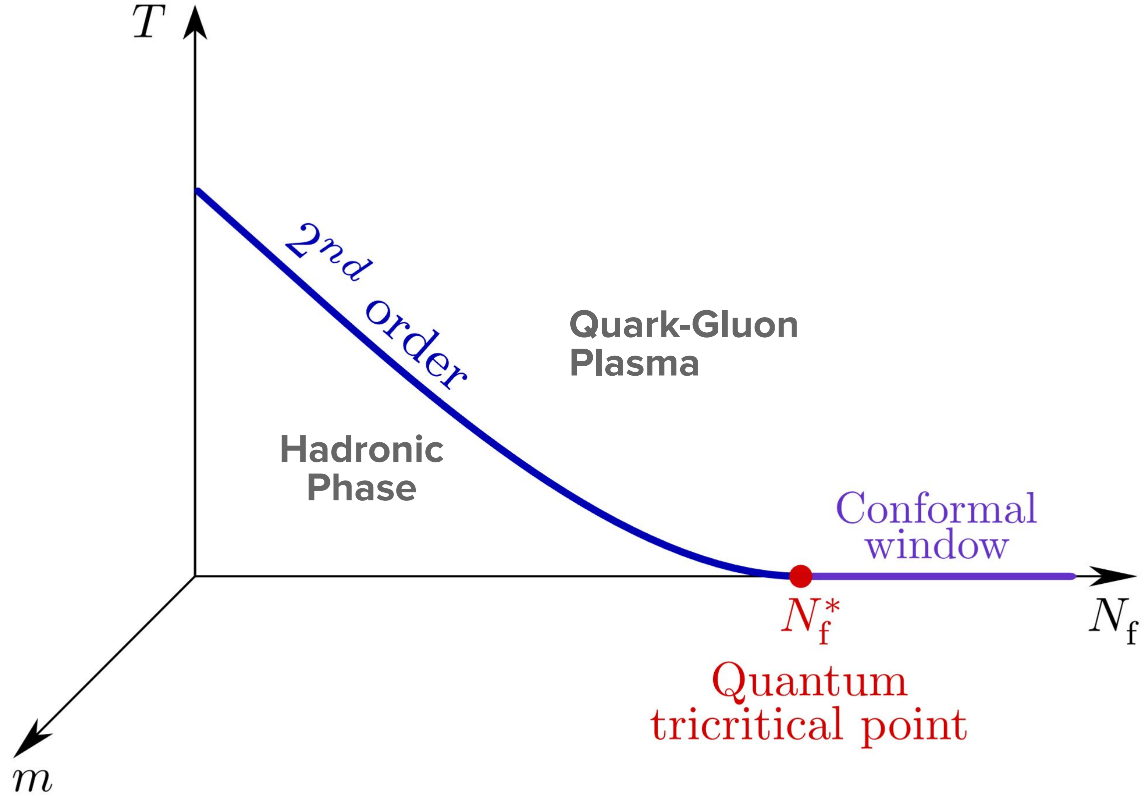
Onset of conformal window Nf*:

$10 \lesssim N_f^* \lesssim 12$ { [Braun, Gies 11] [Lombardo, Pallante, Deuzeman 13]

$8 \lesssim N_f^* \lesssim 9$ [Hasenfratz et al. 23]

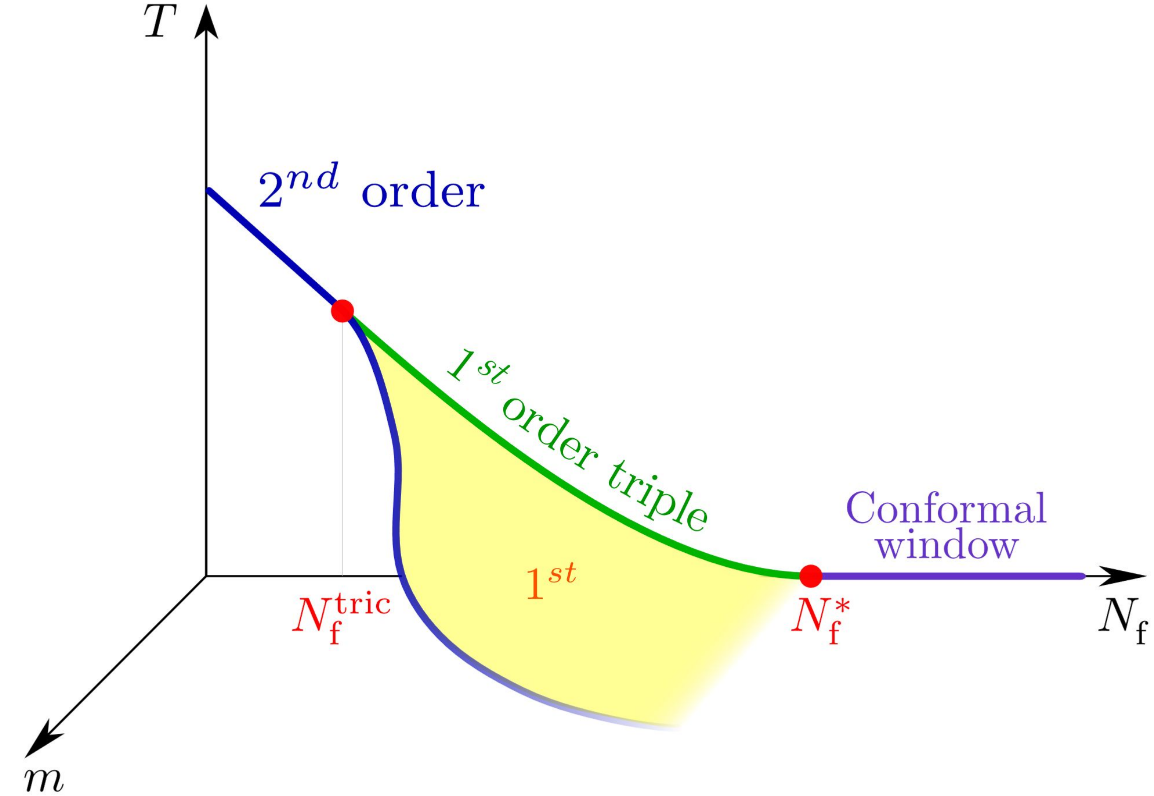
$12 < N_f^* < 14$ [Fodor et al. 18]

Scenario 1



- 2nd order for all N_f
- $N_f^{tric} = N_f^*$

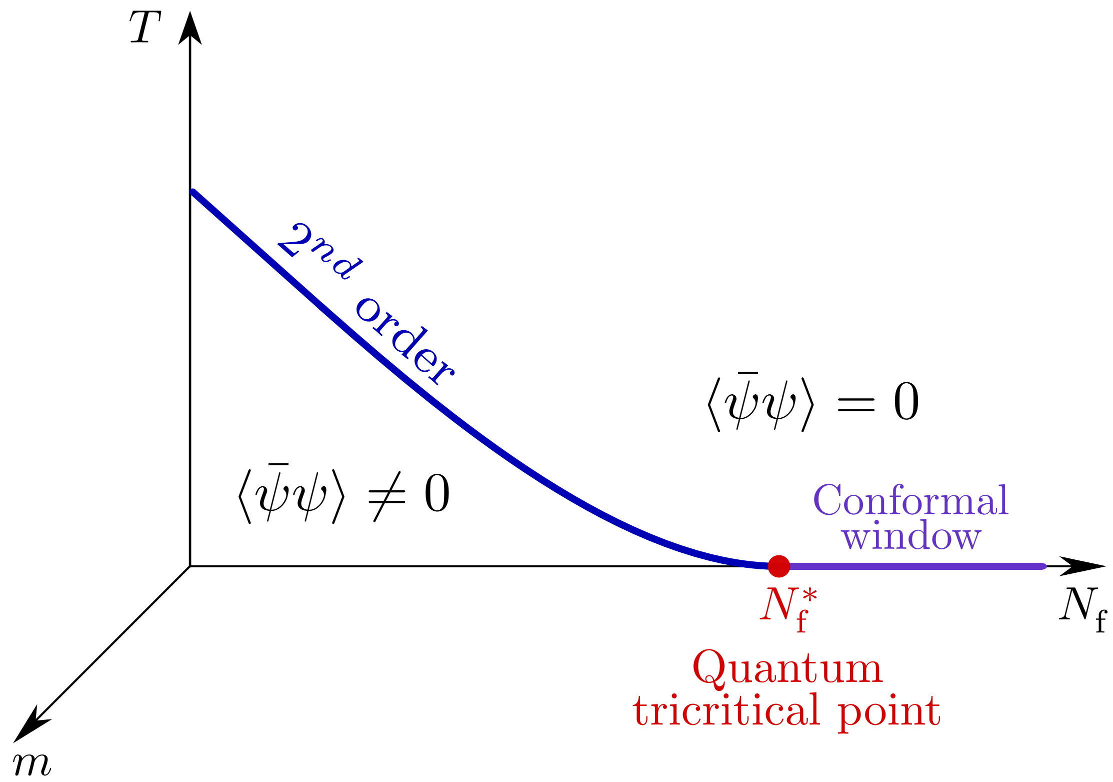
Scenario 2



- 2nd order turns into 1st order at N_f^{tric}
- $6 < N_f^{tric} < N_f^*$

Motivation: a) "Conformalisation of nuclear matter" reported from neutron stars, eqn. of state
b) Physics beyond the Standard Model: "walking technicolour", composite Higgs
c) IR conformality a la Alexandru, Horvath

Difficulties^3



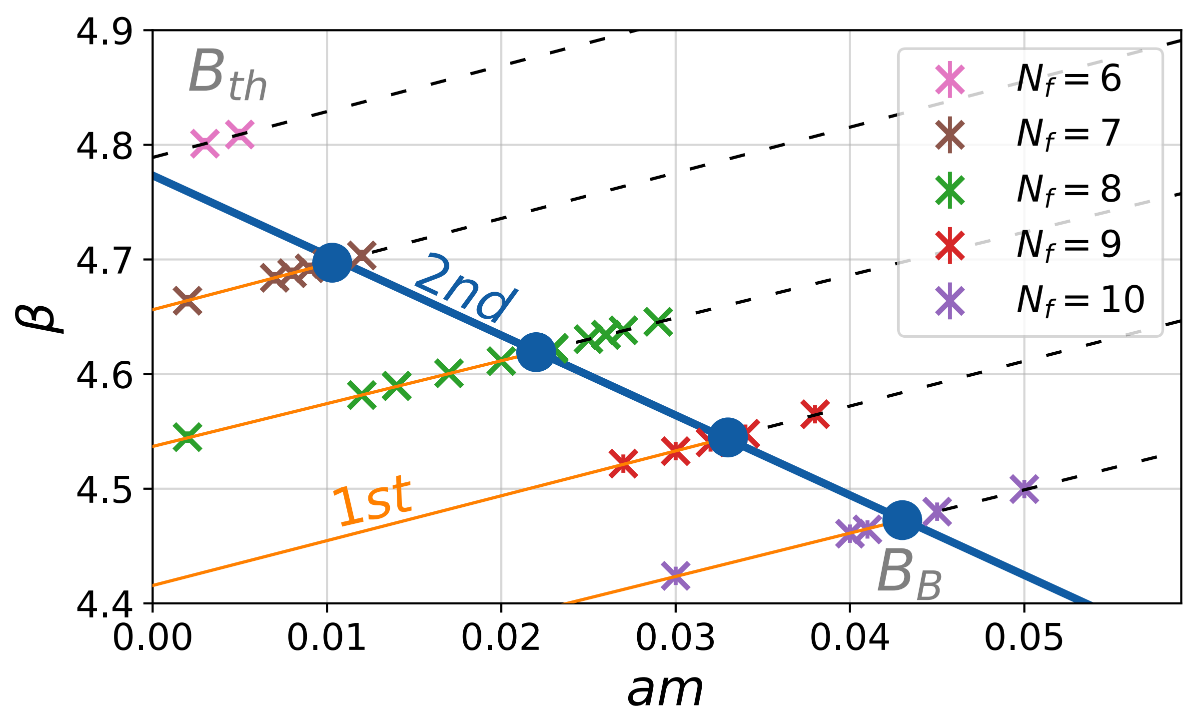
- Conformal window: no symmetry breaking in chiral limit m=0 at T=0
- m=0 cannot be simulated
- T=0 cannot be simulated, $N_\tau = \infty$
- Everything that can be simulated has finite m, T:
  explicit symmetry breaking + transition to restored symmetry at higher T

Obstacle: an unphysical bulk phase + transition

Phase transition at strong gauge coupling, i.e. small $\beta(a) = \frac{6}{g^2(a)}$

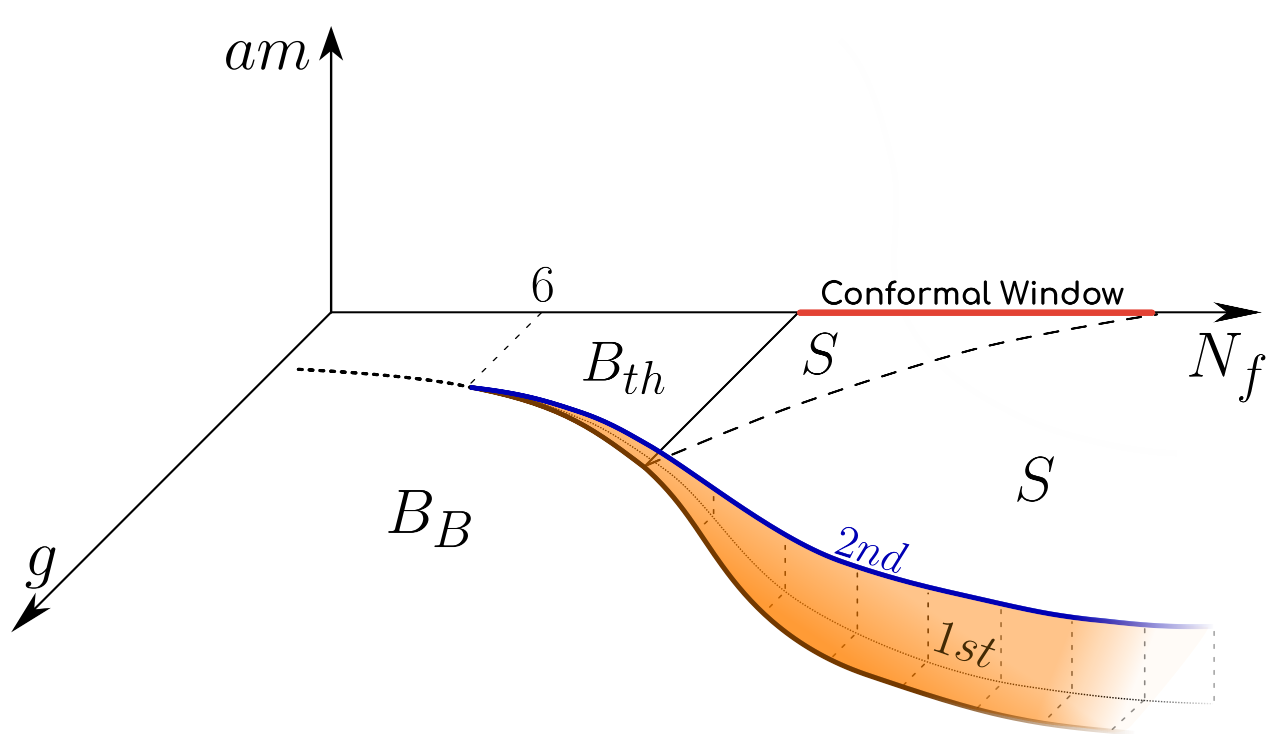
Independent of N_τ ➡ lattice artefact, not connected to continuum

Happens because $T = \frac{1}{aN_\tau}$ approaches zero at too small N_τ



$N_\tau > 8$

T=0 lattice phase diagram

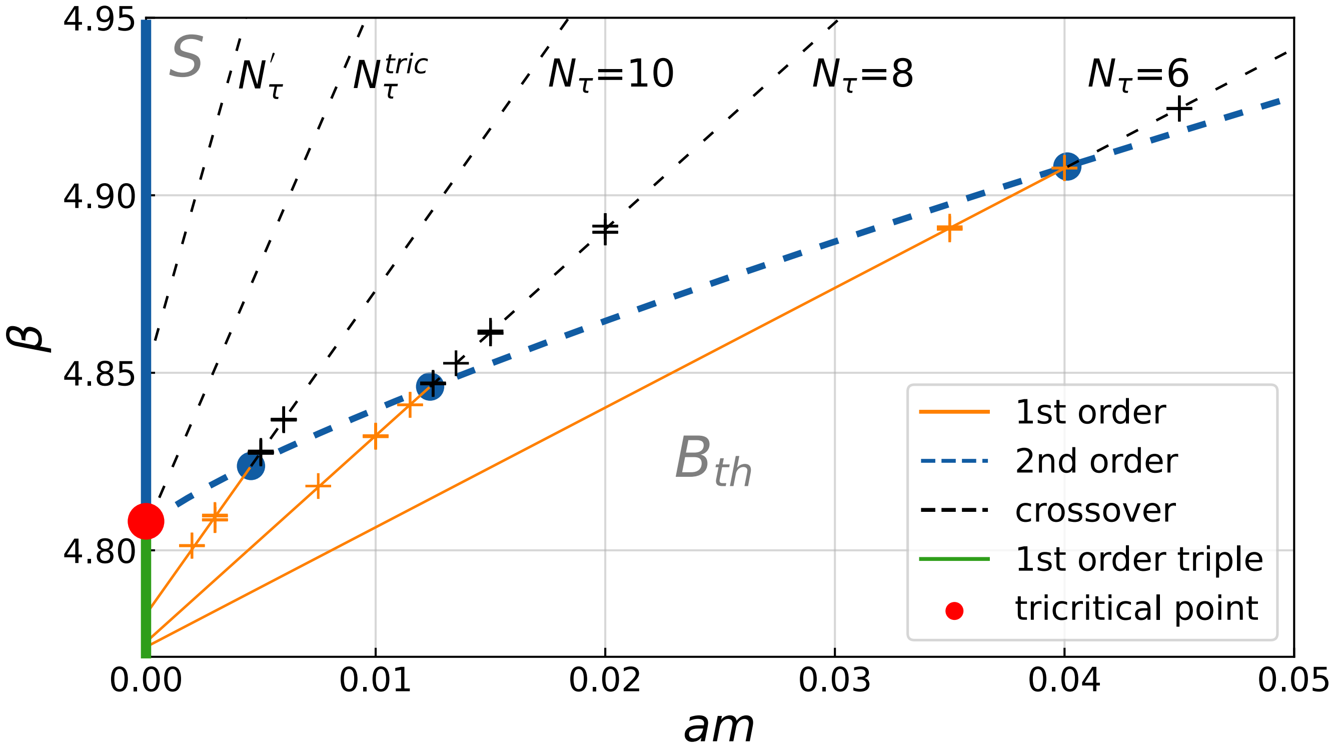


[Kaiser, Klinger, O.P., Schaible, PoS LAT 25]

Latest numerical results

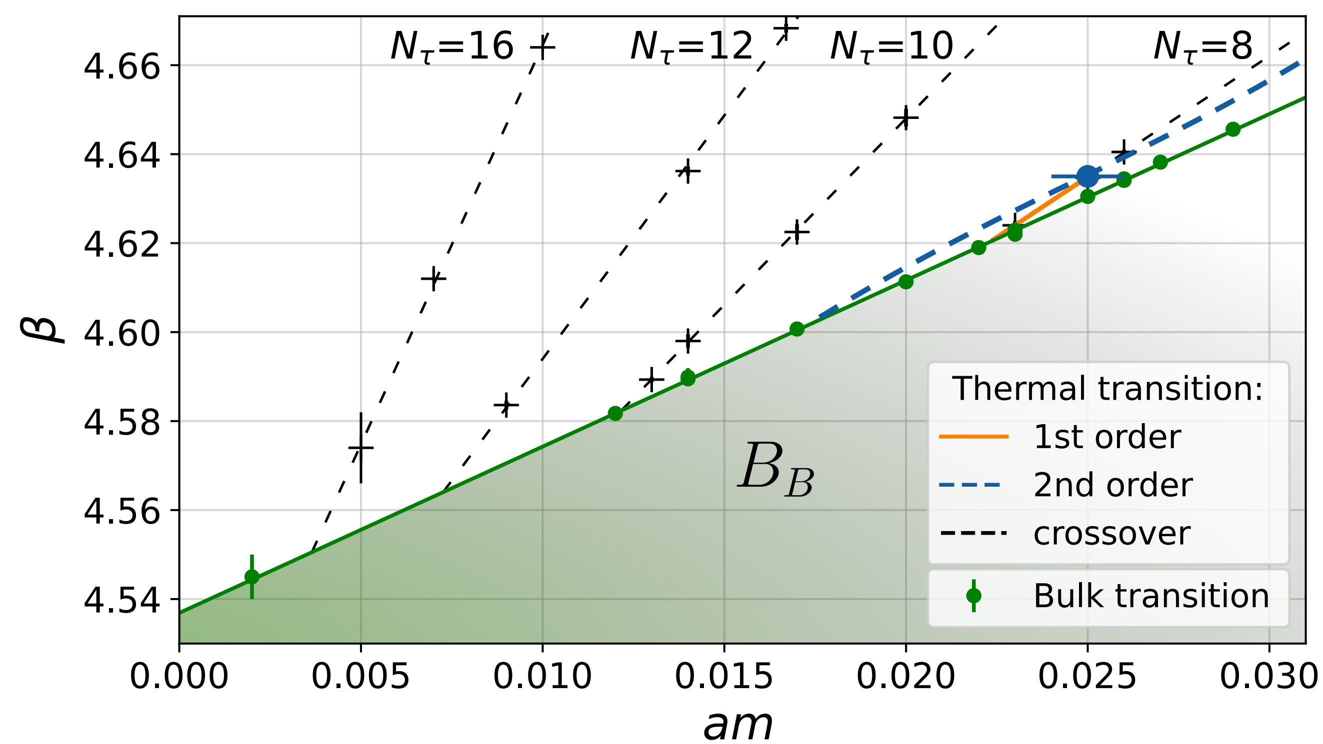
$N_f = 6$

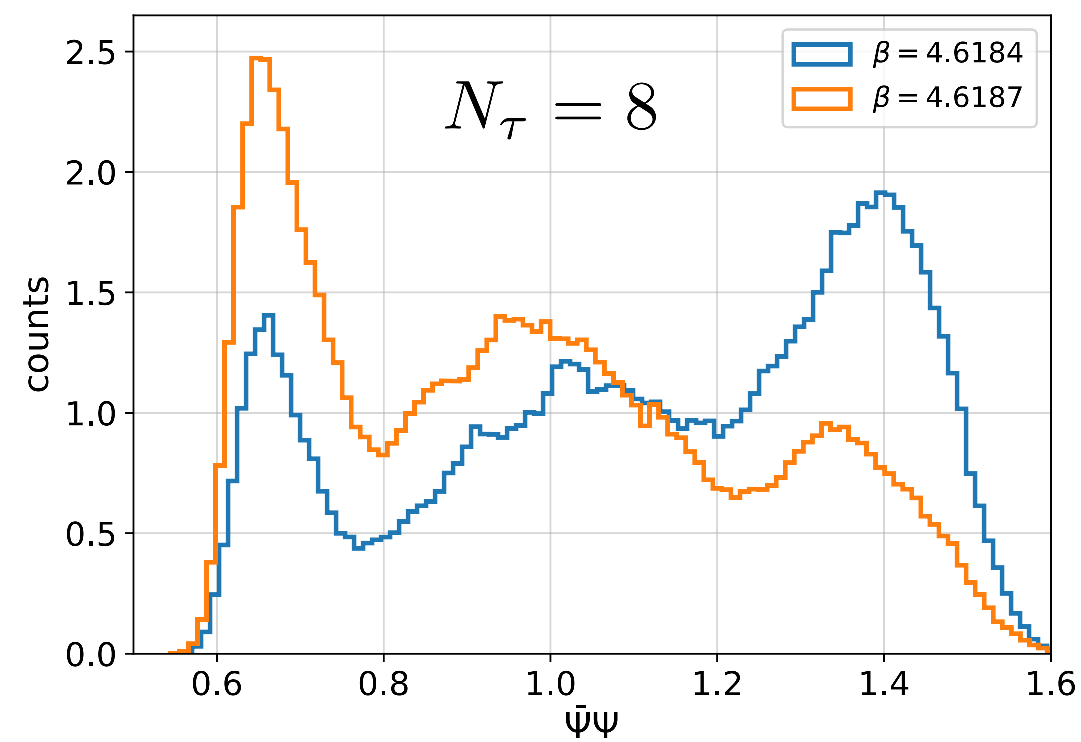
Thermal phase transition



$N_f = 8$

Thermal phase transition + bulk transition





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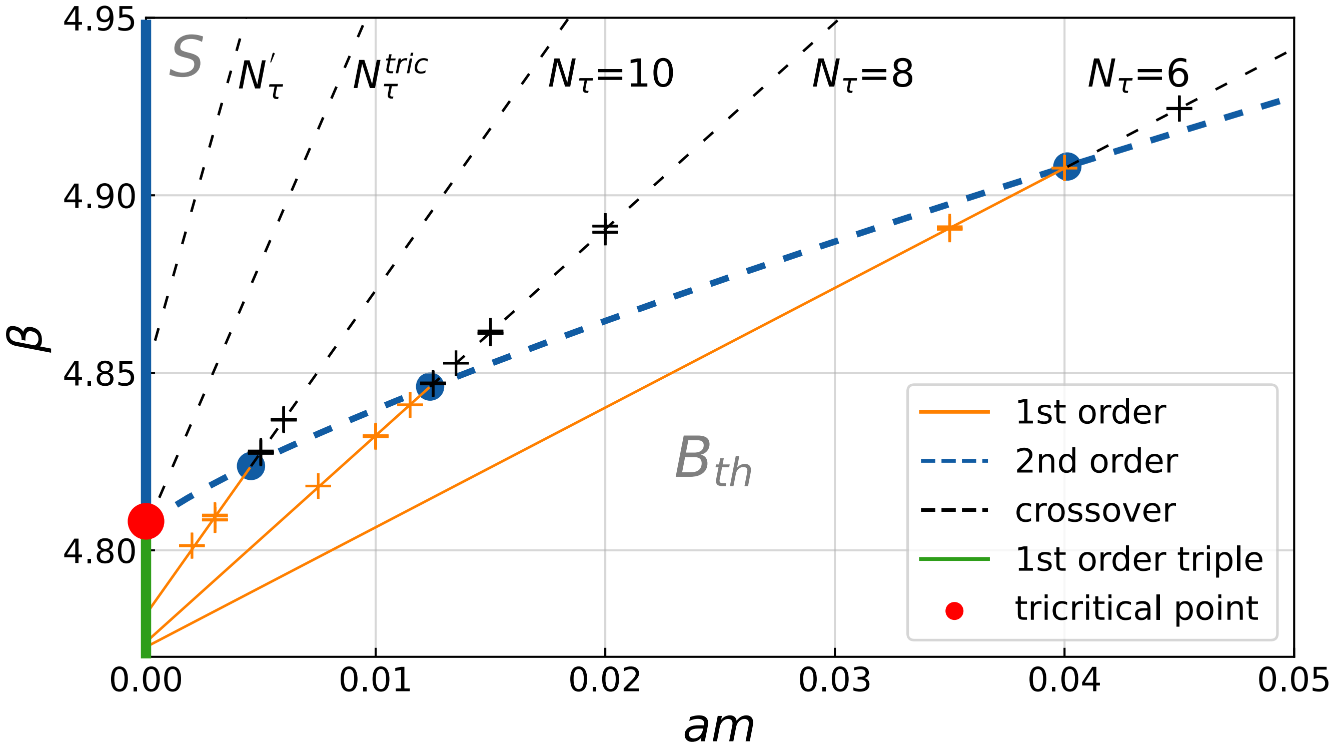
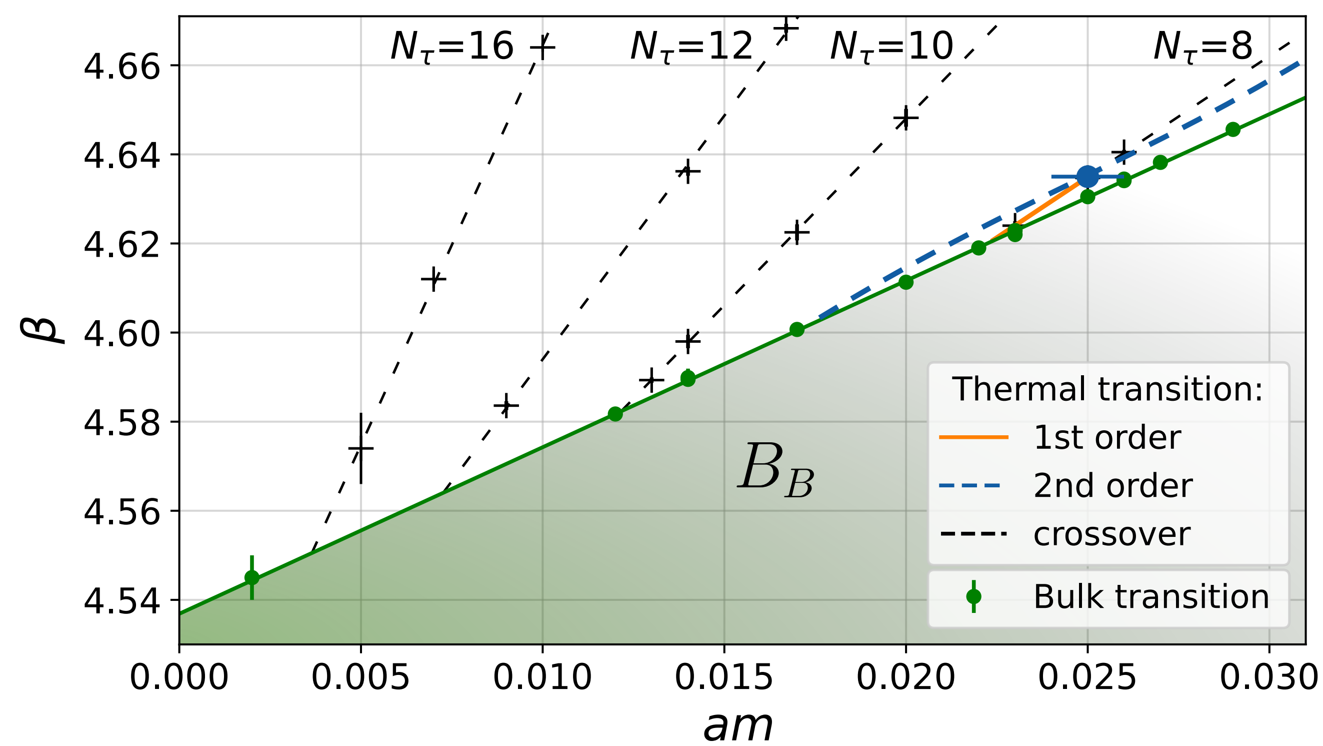


$N_f = 8$

Thermal phase transition + bulk transition



Continuum limit of 2.O. chiral transition exists

$$a \to 0$$
$$\beta(a) \to \infty$$
$$N_\tau \to \infty$$

No chiral transition exists in continuum!

Our results suggest: $7 < N_f^* < 8$

to be proven with some larger N_τ

Effective d.o.f.: from spectral functions

Usual starting point:

temporal lattice correlators

$$C_\Gamma(\tau, \boldsymbol{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau,\omega)\rho_\Gamma(\omega,\boldsymbol{p}) \ ,$$

$$K(\tau,\omega) = \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \ .$$

- Inversion problem on finite number of data points not well-defined
- Statistical approaches to find "most likely" spectral function:

  Maximum entropy and other Bayesian methods, Backus-Gilbert method,….
- Provide constraints/bias by analytic results, models, …
- Reviews: [Asakawa, Hatsuda, Nakahara, PPNP 01
  Meyer, PoS INPC 16
  Spriggs et al., EPJ Web Conf. 22
  …]

Spectral functions from “0th principles”

J. Bros + D. Buchholz: finite T QFT a la Wightman

- Towards a relativistic KMS condition, NPB 94
- Axiomatic analyticity properties and representations of particles in finite T QFT, Ann. Inst. Poincare Phys. Theor. 96
- Asymptotic dynamics of thermal quantum fields, NPB 02

Rigorous result: see also [Nair, Pisarski PRD 25]

$$\rho_{\rm PS}(p_0,\vec{p}) = \int_0^\infty ds \int \frac{d^3\vec{u}}{(2\pi)^2}\, \epsilon(p_0)\, \delta\left(p_0^2 - (\vec{p}-\vec{u})^2 - s\right) \widetilde{D}_\beta(\vec{u},s)$$

Exact, goes to Källen-Lehmann representation for $T \to 0$ ↑ thermal spectral density

➡ Relation between spatial correlators and thermal spectral density

$$C^s_{PS}(z) = \frac{1}{2}\int_0^\infty ds \int_{|z|}^\infty dR\, e^{-R\sqrt{s}} D_\beta(R,s)$$

[Lowdon, O.P., JHEP 22]

5 Thermoparticles

[D. Buchholz, in
"Workshop on Math. Phys towards the 21st centrury" 1993]

Let us turn now to the analysis of the particle-like constituents of thermal equilibrium states. As will become clear, these entities have to be distinguished from quasiparticles, describing the collective excitations of thermal states, and also from the other particle structures analyzed so far. We therefore propose to call them *thermoparticles*.[10]

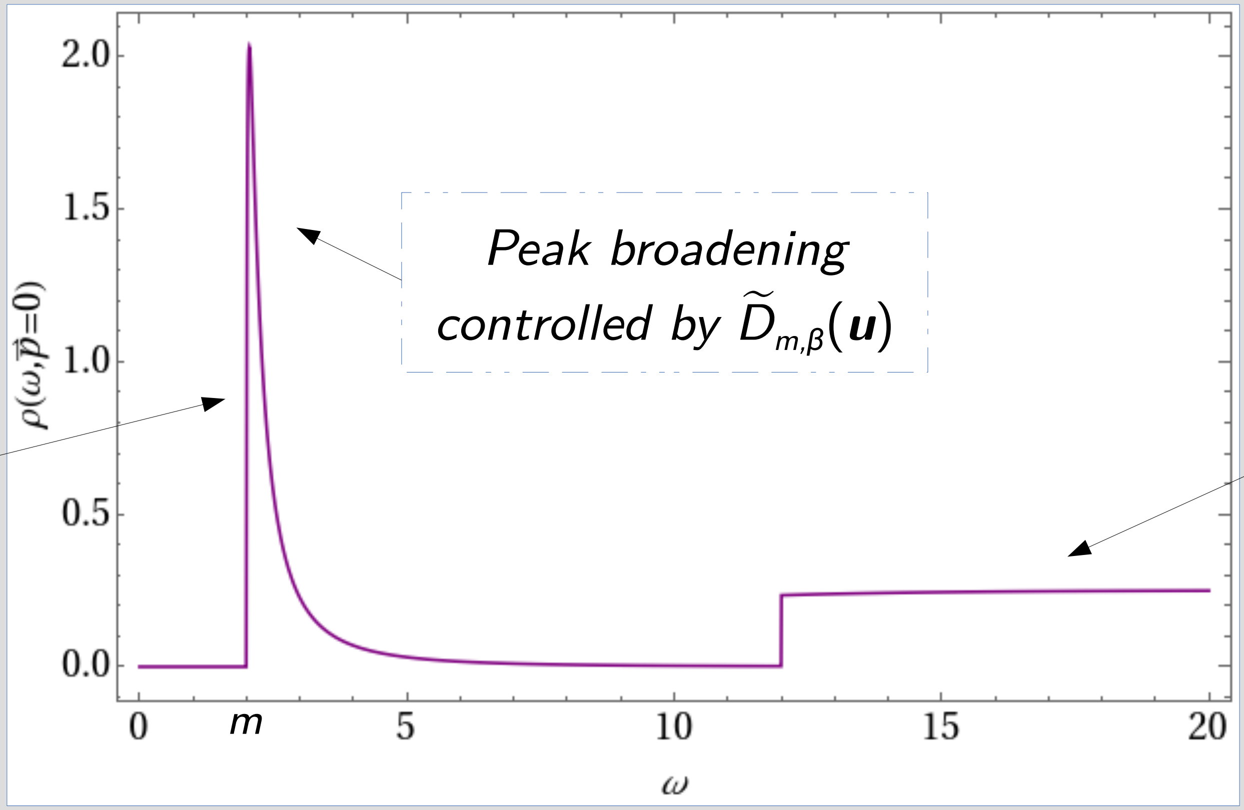
$$\widetilde{D}_\beta(\vec{u}, s) = \widetilde{D}_{m,\beta}(\vec{u})\,\delta(s - m^2) + \widetilde{D}_{c,\beta}(\vec{u}, s)$$

Thermoparticle component

"Damping factor"

Causes $T = 0$ mass pole m to be *<u>screened</u>* *by thermal effects*

Dominantes asymptotic times



Continuous component

Fixes T-dependence of continuous spectral contributions

Multi-particle, collective, Landau damping

Dominance of particle states at low T

- General spectral decomposition of spatial correlators with eigenstates of H

$$C_\Gamma(\tau, \vec{x}) = \frac{1}{\mathcal{Z}} \sum_{m,n} |\langle m|O_\Gamma(0,\vec{x})|n\rangle|^2 \, e^{-\tau E_n} e^{-(\beta-\tau)E_m}$$

Vacuum states still dominant for low temperatures

- Vector+axial vector correlators in the chiral limit using PCAC, $\epsilon = T^2/(6 f_\pi^2)$

[Dey, Eletsky, Ioffe PLB 90]

$$C_V(p,T) = (1-\epsilon) C_V(p,0) + \epsilon C_A(p,0)$$
$$C_A(p,T) = (1-\epsilon) C_A(p,0) + \epsilon C_V(p,0)$$

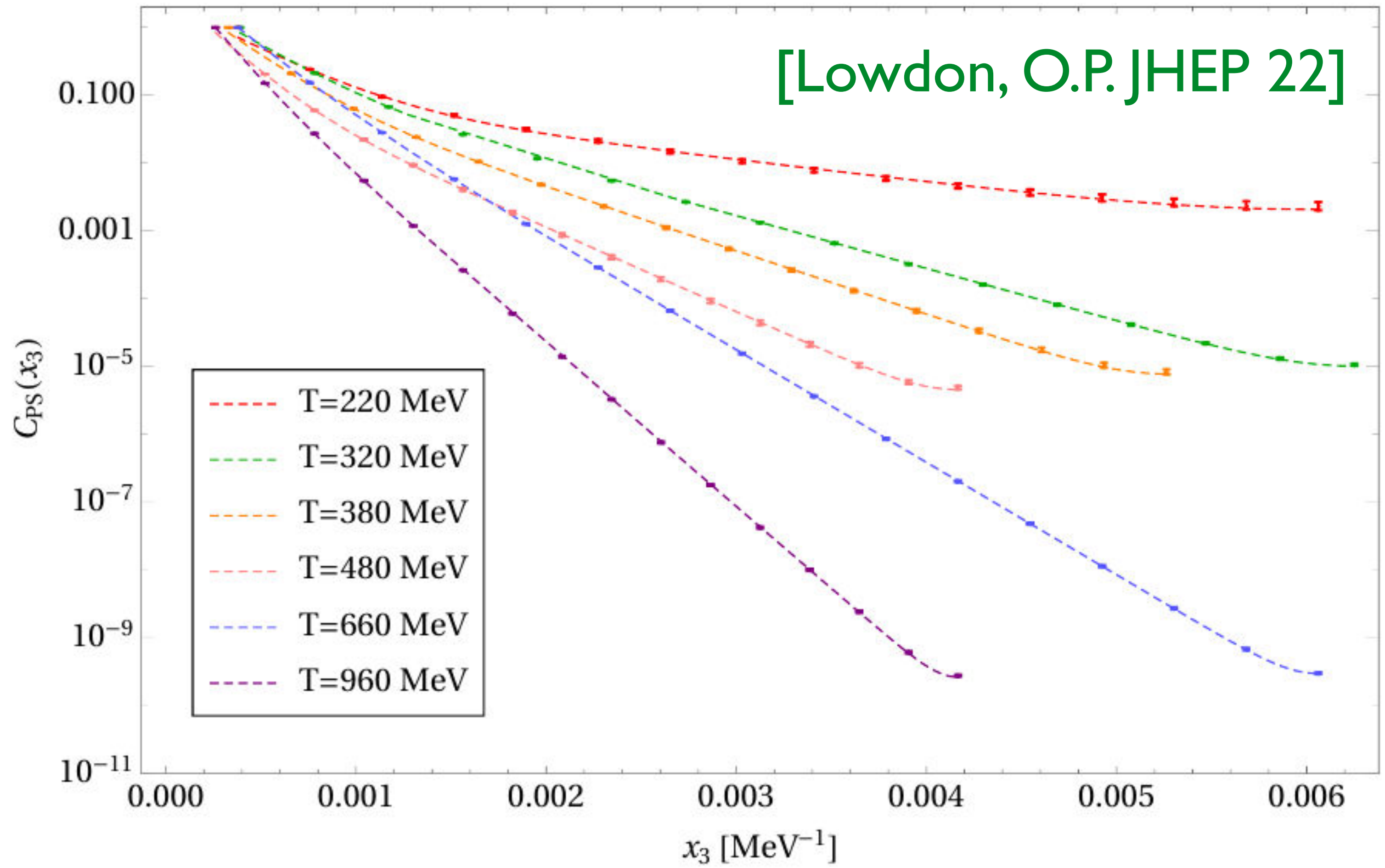
Vacuum pole structure of correlators reflected in finite T correlators

The pion spectral function from spatial correlators

Lattice input: $$C^a_{\rm PS}(x_3) = \frac{1}{N}\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 \int_{-\frac{\beta}{2}}^{\frac{\beta}{2}} d\tau \, \langle \Omega_\beta | \mathcal{O}^a_{\rm PS}(\tau,\vec{x}) \mathcal{O}^{a\,\dagger}_{\rm PS}(0) | \Omega_\beta \rangle$$

Data from:

[Rohrhofer et al., PRD 19]



Excellent two-expontential fits (sceening masses) obtained for entire range, all T:

$$C_{\rm PS}(x_3) = A e^{-m_\pi^{\rm scr} x_3} + A e^{-m_\pi^{\rm scr}(L-x_3)} + B e^{-m_{\pi *}^{\rm scr} x_3} + B e^{-m_{\pi *}^{\rm scr}(L-x_3)}$$

Damping factors for pion + first excitation: $D_{m_{\pi^{(*)}},\beta} = \alpha_{\pi^{(*)}} e^{-\gamma_{\pi^{(*)}} x_3}$

These generate the spatial correlators

$$C(x_3) = \sum_{i=\pi,\pi^*} C_i(x_3)\ , \qquad C_i(x_3) = \frac{\alpha_i}{2(m_i+\gamma_i)} e^{-(m_i+\gamma_i)|x_3|}$$

Analytic relation between screening masses, vacuum masses and damping parameters:

$$m_i^{\mathrm{scr}}(T) = m_i + \gamma_i(T), \qquad i = \pi,\ \pi^*$$

Check zero temperature limit: $C_i(x_3) \xrightarrow{T\to 0} \dfrac{\alpha_i(T=0)}{2m_i} e^{-m_i|x_3|}$

Damping parameters must vanish, Lorentz invariance restored!

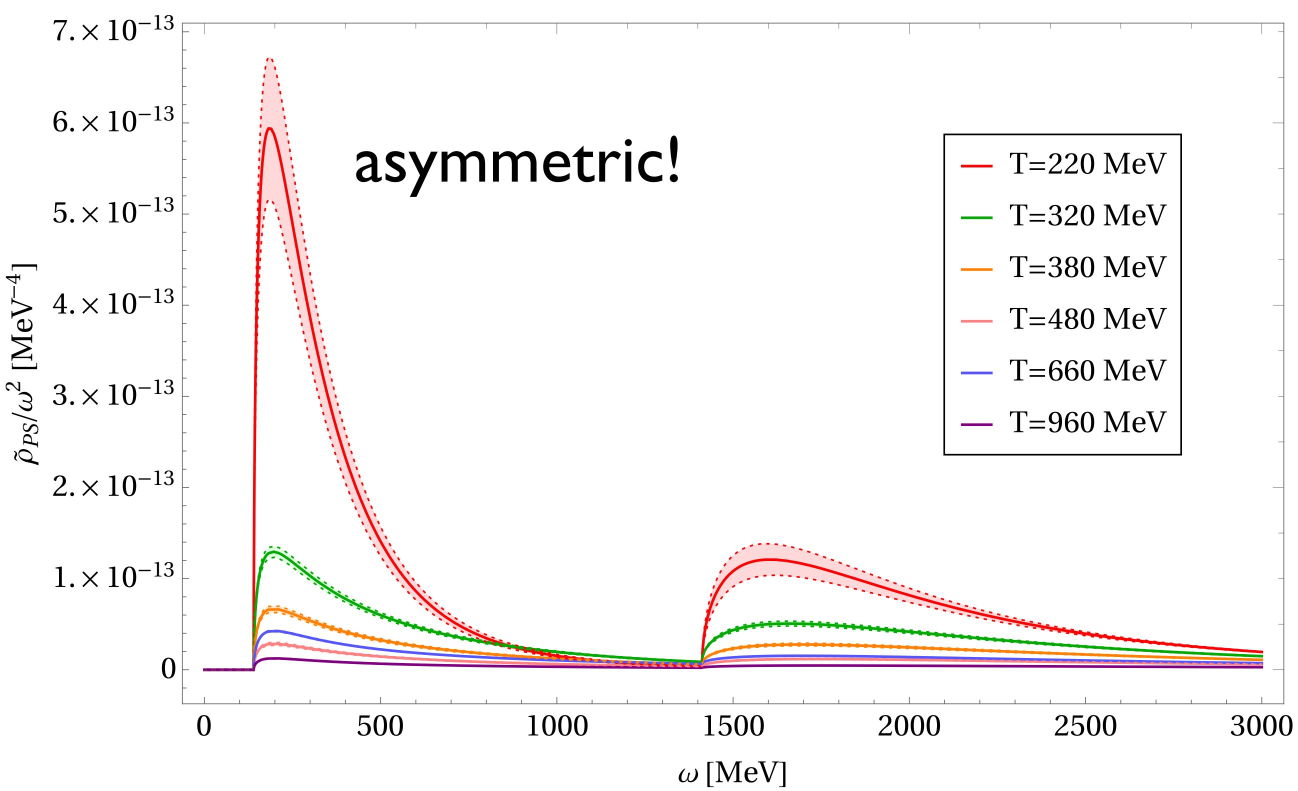
From damping factors to spectral functions

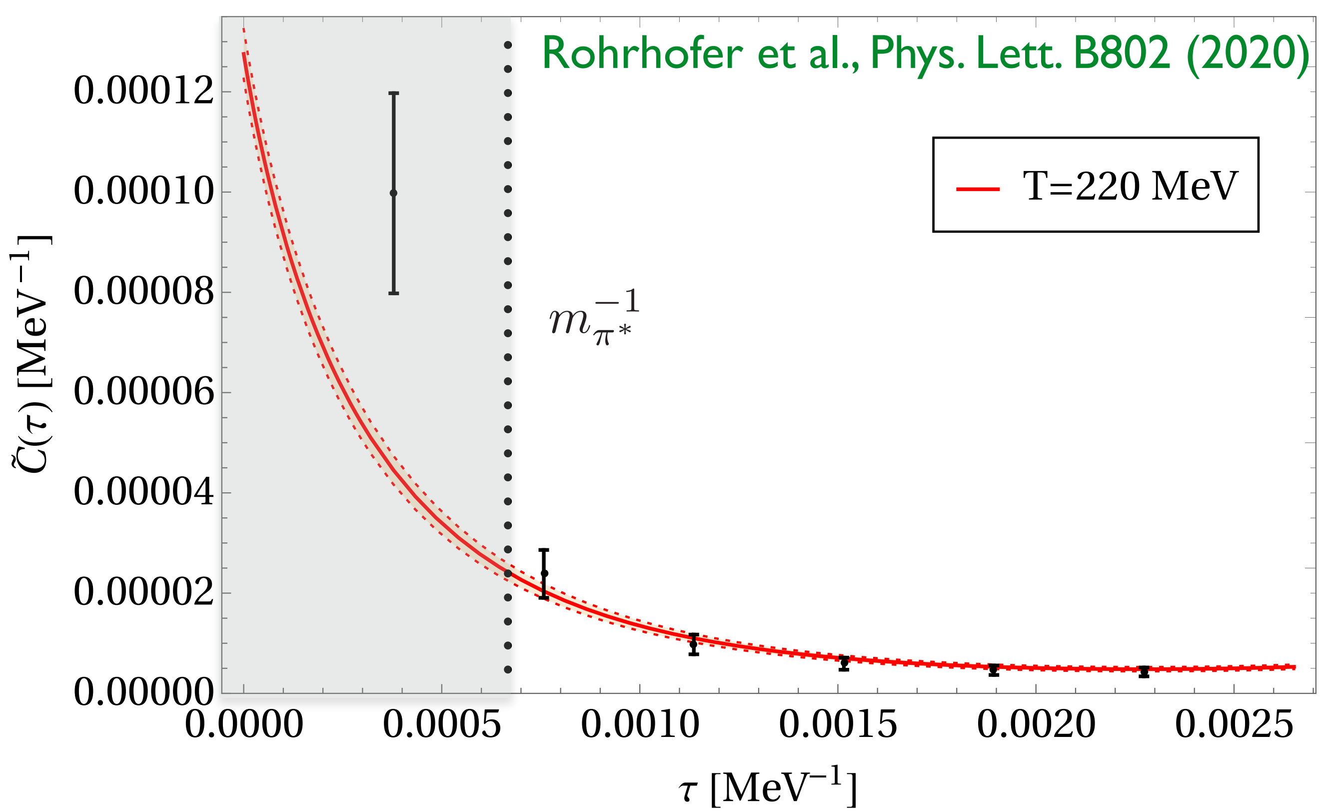
$$D_\beta(\vec{x}, s) = D_{m_\pi,\beta}(\vec{x})\,\delta(s - m_\pi^2) + D_{m_{\pi*},\beta}(\vec{x})\,\delta(s - m_{\pi*}^2)$$

Continuum contributions neglected (no multi-particle scattering states, Landau damping)

$$\rho_{\rm PS}(\omega, \vec{p} = 0) = \epsilon(\omega)\left[\theta(\omega^2 - m_\pi^2)\,\frac{4\,\alpha_\pi\,\gamma_\pi\sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma_\pi^2)^2} + \theta(\omega^2 - m_{\pi*}^2)\,\frac{4\,\alpha_{\pi*}\,\gamma_{\pi*}\sqrt{\omega^2 - m_{\pi*}^2}}{(\omega^2 - m_{\pi*}^2 + \gamma_{\pi*}^2)^2}\right]$$

spectral function ⟶ predict temporal correlator, compare with data





Comparison thermoparticle vs. plasmon ansatz

Bros+Buchholz

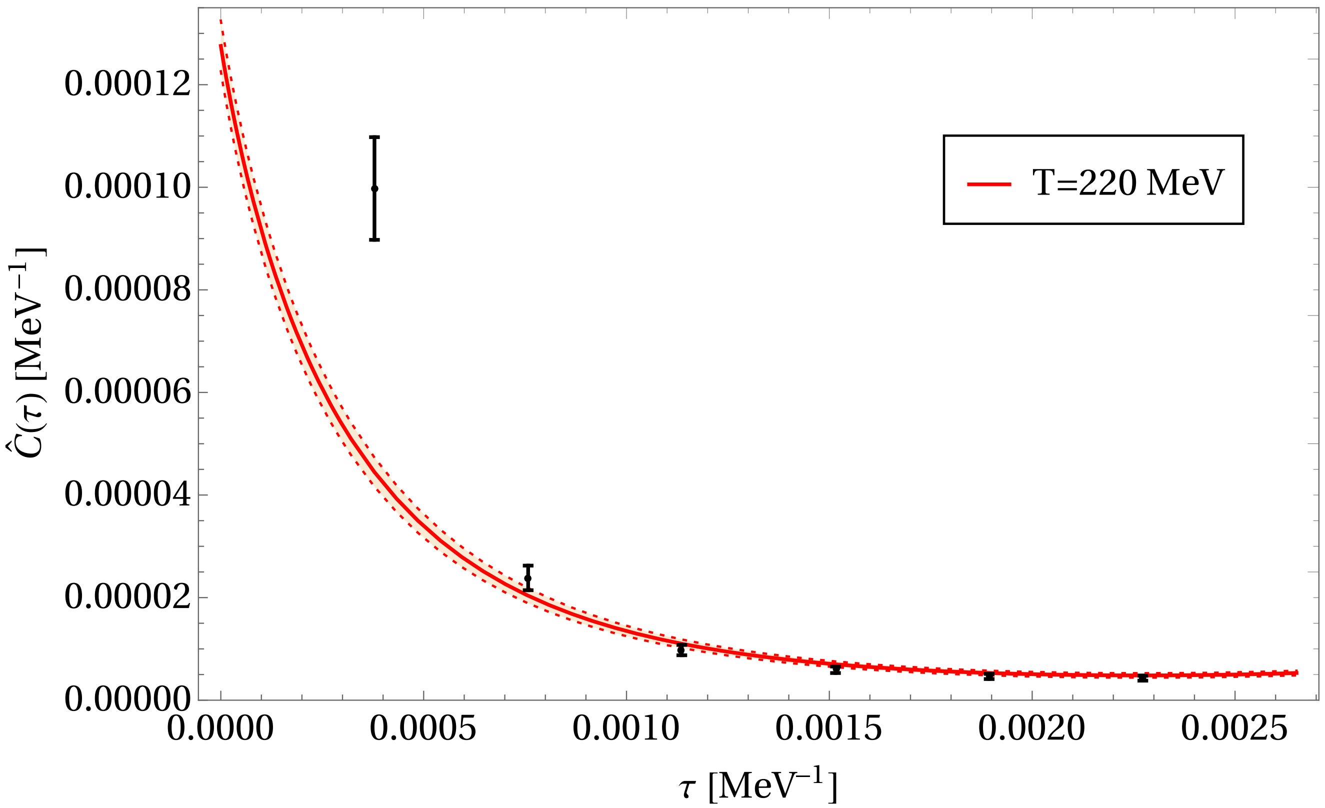
Perturbative plasmon: Breit-Wigner

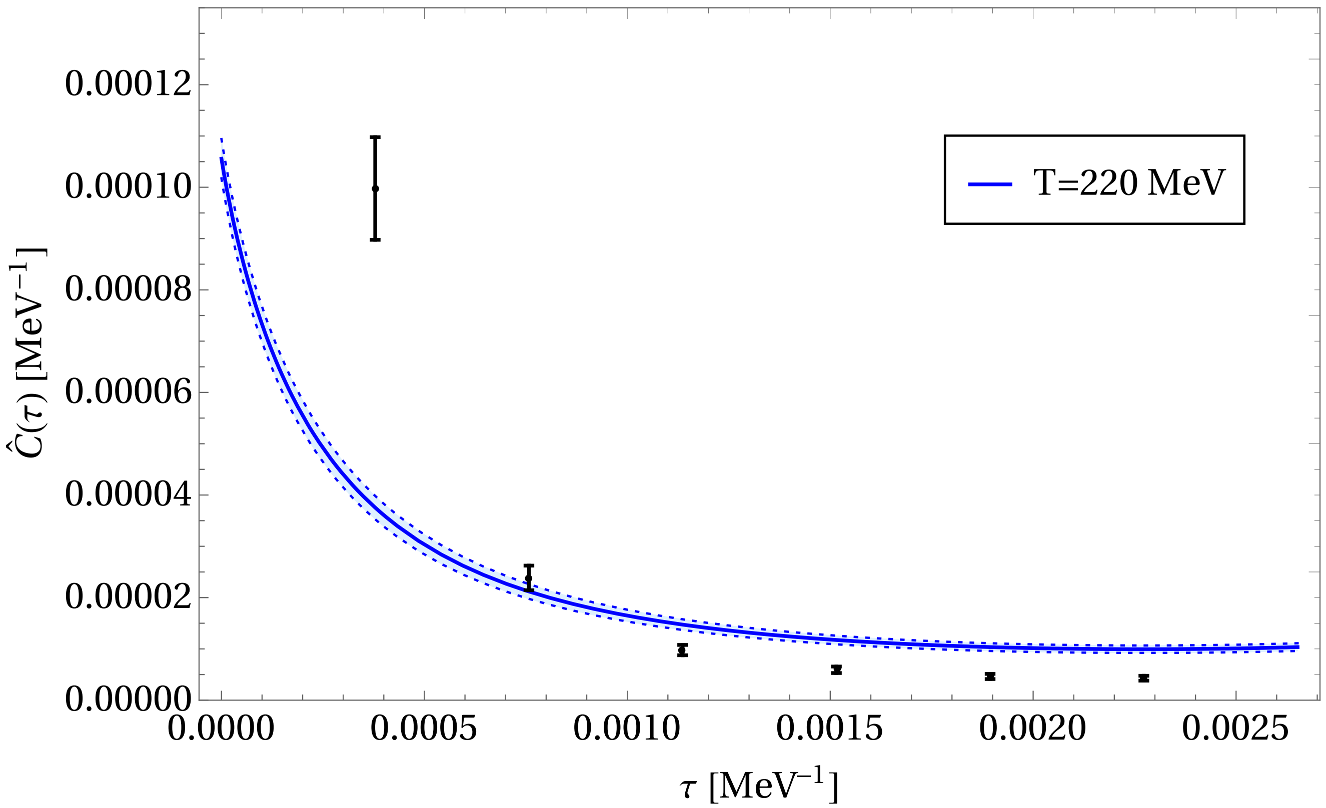
Both fit spatial correlator

$$\rho_{\mathrm{PS}}(\omega, \mathbf{p}=0) = \epsilon(\omega)\left[\theta(\omega^2 - m_\pi^2)\,\frac{4\,\alpha_\pi\,\gamma_\pi\sqrt{\omega^2 - m_\pi^2}}{(\omega^2 - m_\pi^2 + \gamma_\pi^2)^2} + \theta(\omega^2 - m_{\pi^*}^2)\,\frac{4\,\alpha_{\pi^*}\,\gamma_{\pi^*}\sqrt{\omega^2 - m_{\pi^*}^2}}{(\omega^2 - m_{\pi^*}^2 + \gamma_{\pi^*}^2)^2}\right]$$

$$\rho_{PS}^{BW}(\omega, \mathbf{p}=0) = \frac{4\alpha_\pi \omega \Gamma_\pi}{(\omega^2 - m_\pi^2 - \Gamma_\pi^2)^2 + 4\omega^2\Gamma_\pi^2} + \frac{4\alpha_\pi^* \omega \Gamma_{\pi^*}}{(\omega^2 - m_{\pi^*}^2 - \Gamma_{\pi^*}^2)^2 + 4\omega^2\Gamma_{\pi^*}^2}$$

Predicted temporal correlators, compared with data:





Thermoparticle damping $\neq$ Breit-Wigner width

Breit-Wigner: describes unstable resonance in vacuum, Lorentz frame independent!

Γ real time decay width

Thermoparicle: describes thermally modified stable particle, does NOT decay, frame dependent

γ thermal damping, modifies spatial decay

Thermoparticle propagator:

$$\widetilde{G}^{\pi}_{\beta}(k_0,\vec{p}) = \frac{\alpha}{|\vec{p}|^2 - k_0^2 + m_\pi^2 + \gamma_\pi^2 + 2\gamma_\pi\sqrt{m_\pi^2 - k_0^2}}. \qquad k_0 = p_0 \pm i\epsilon$$

Not dominated by simple poles, instead: branch points!

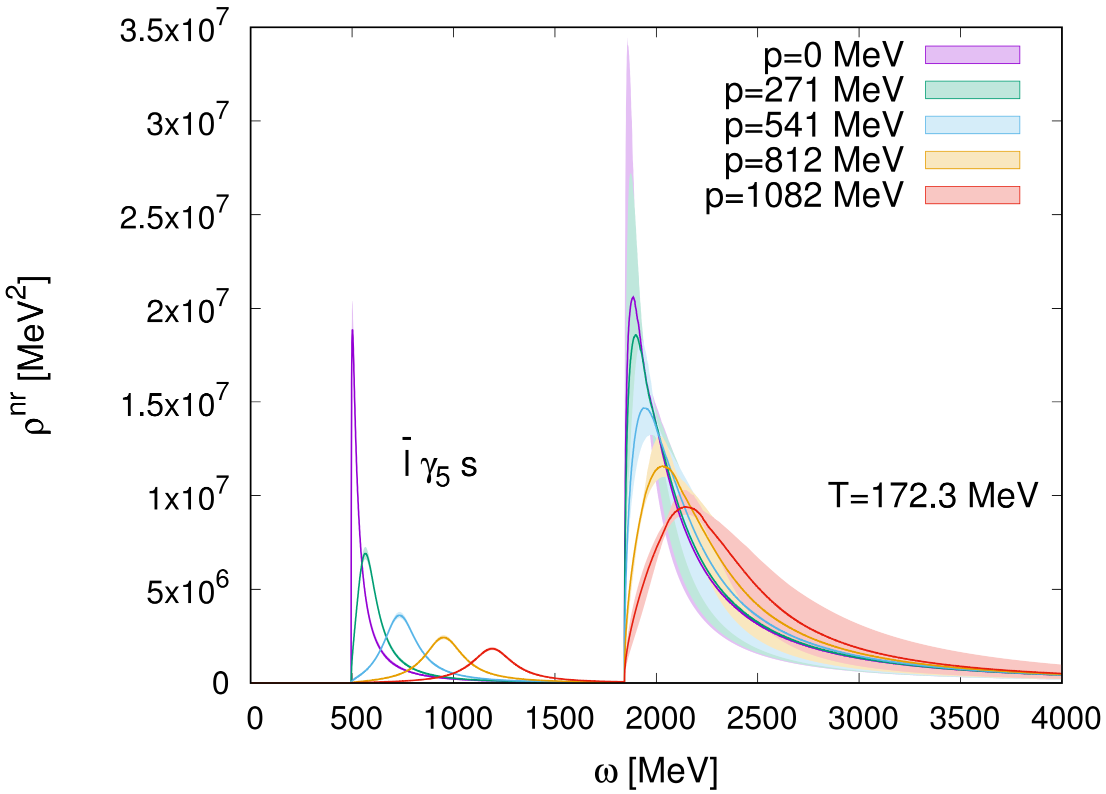
Same analysis for other pseudo-scalars

[Bala, Kaczmarek, Lowdon, O.P., Ueding, JHEP 24]

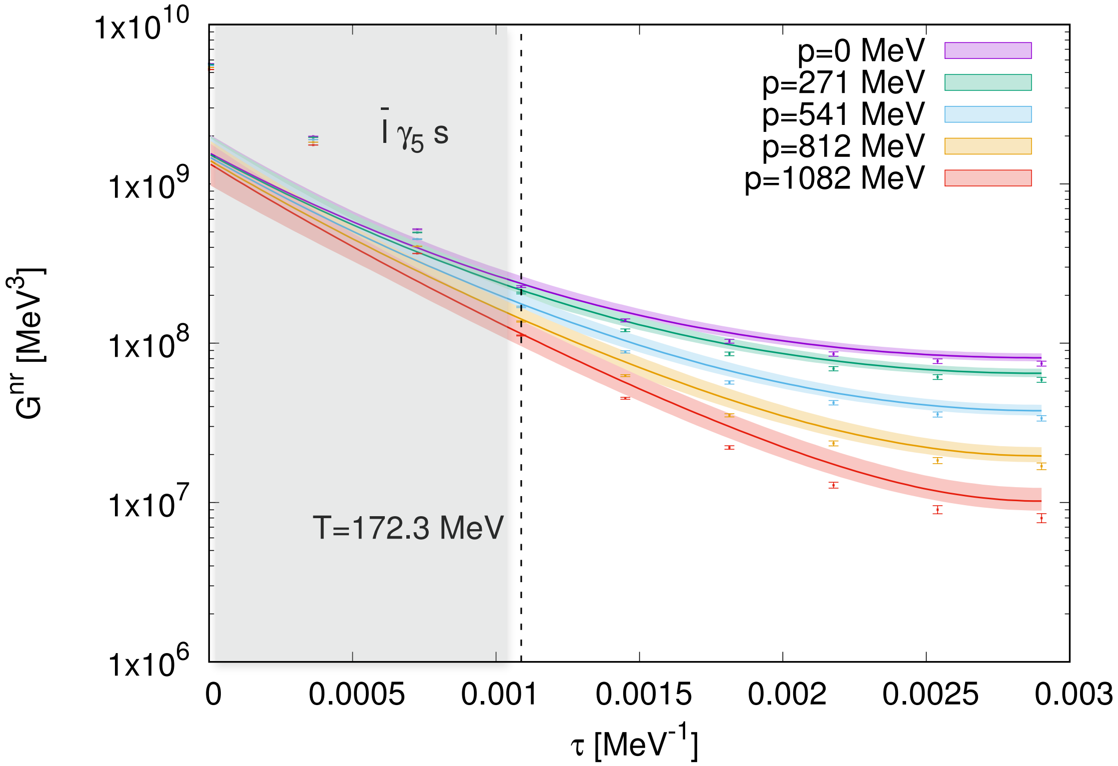
$N_f = 2+1$ HISQ sea + domain wall valence quarks, physical masses on $64^3 \times 16$

Goal: analyse all scalar and pseudo-scalar correlators, here: $\bar{l}s$ - channel (Kaon)

spectral function from z-correlator



predicted + measured t-correlator

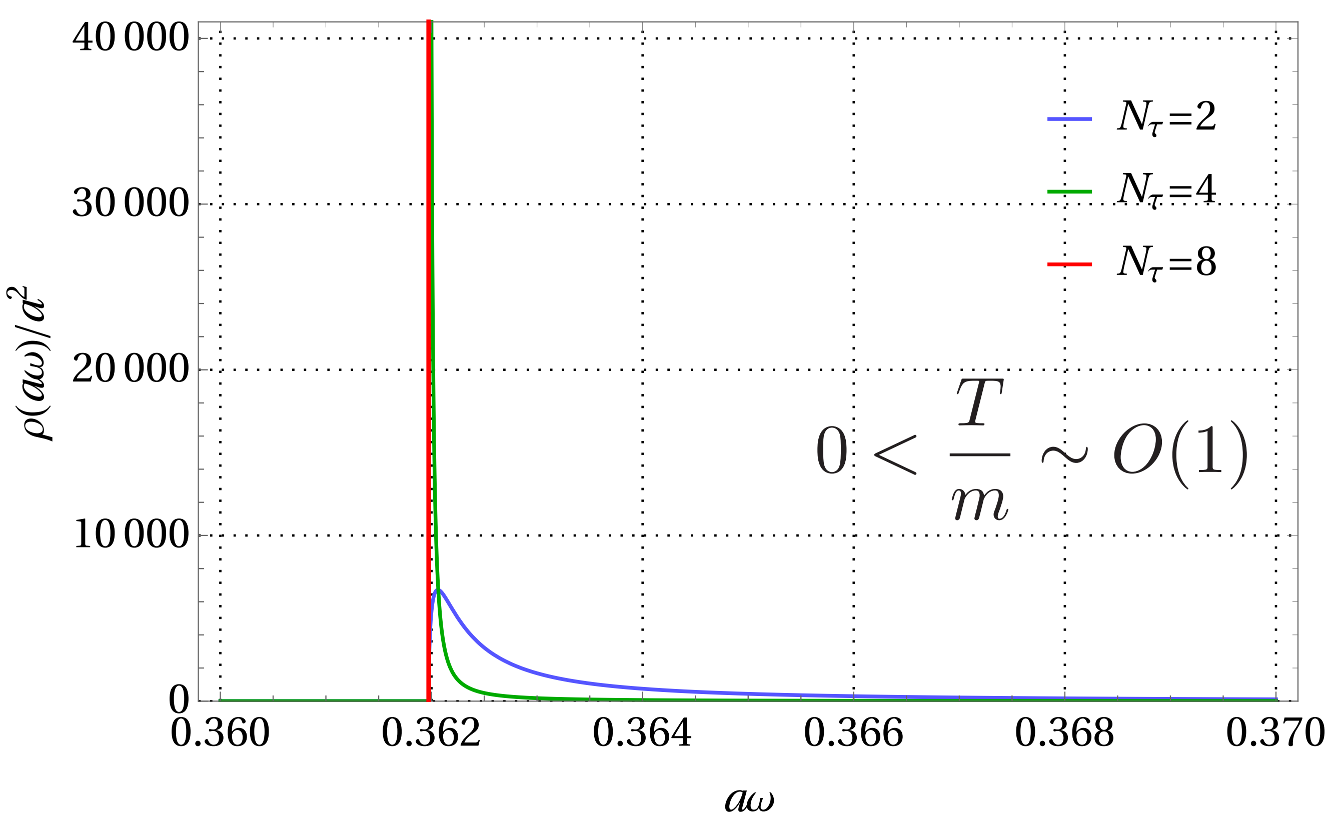


Increasing momentum:

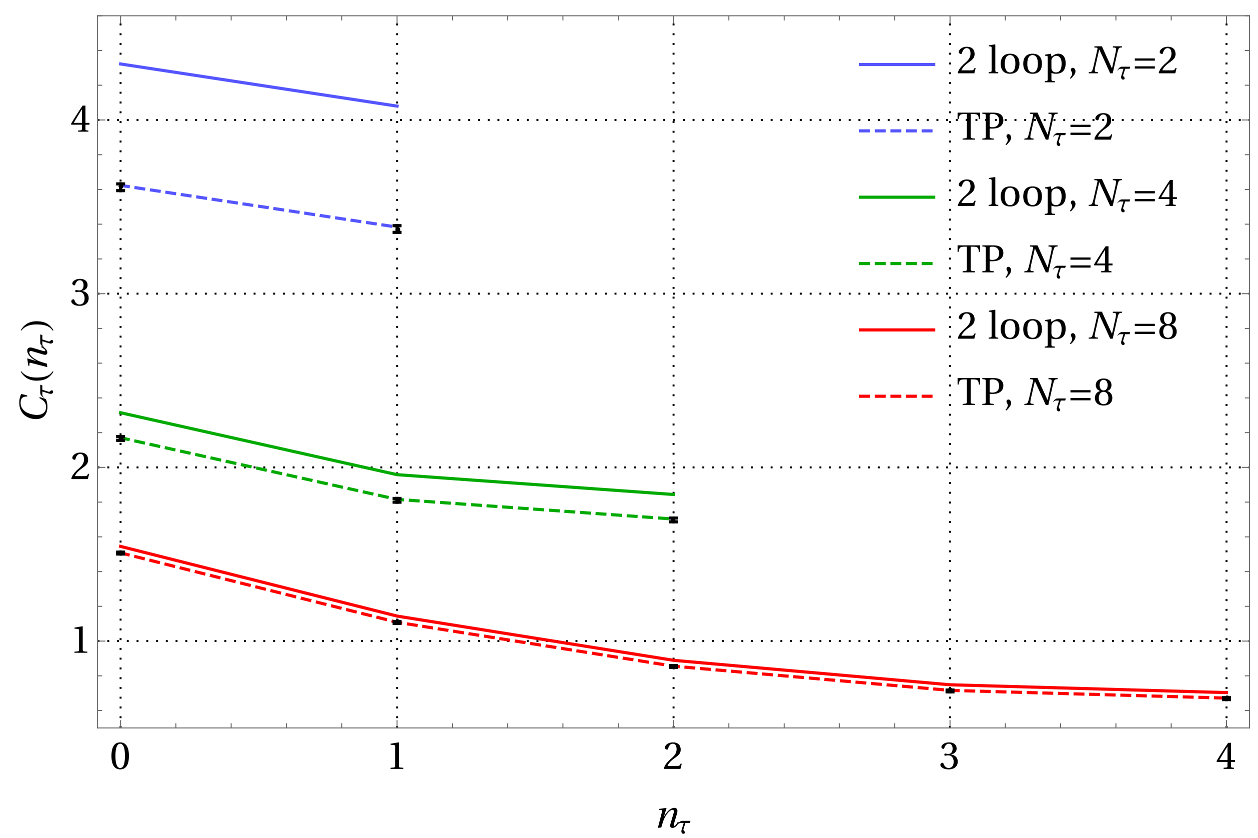
Approximation deteriorates, continuum states increasingly important!

Thermoparticle check: massive ϕ^4-theory [Lowdon, O.P., JHEP 24]

spectral function from z-correlator



predicted + measured t-correlator

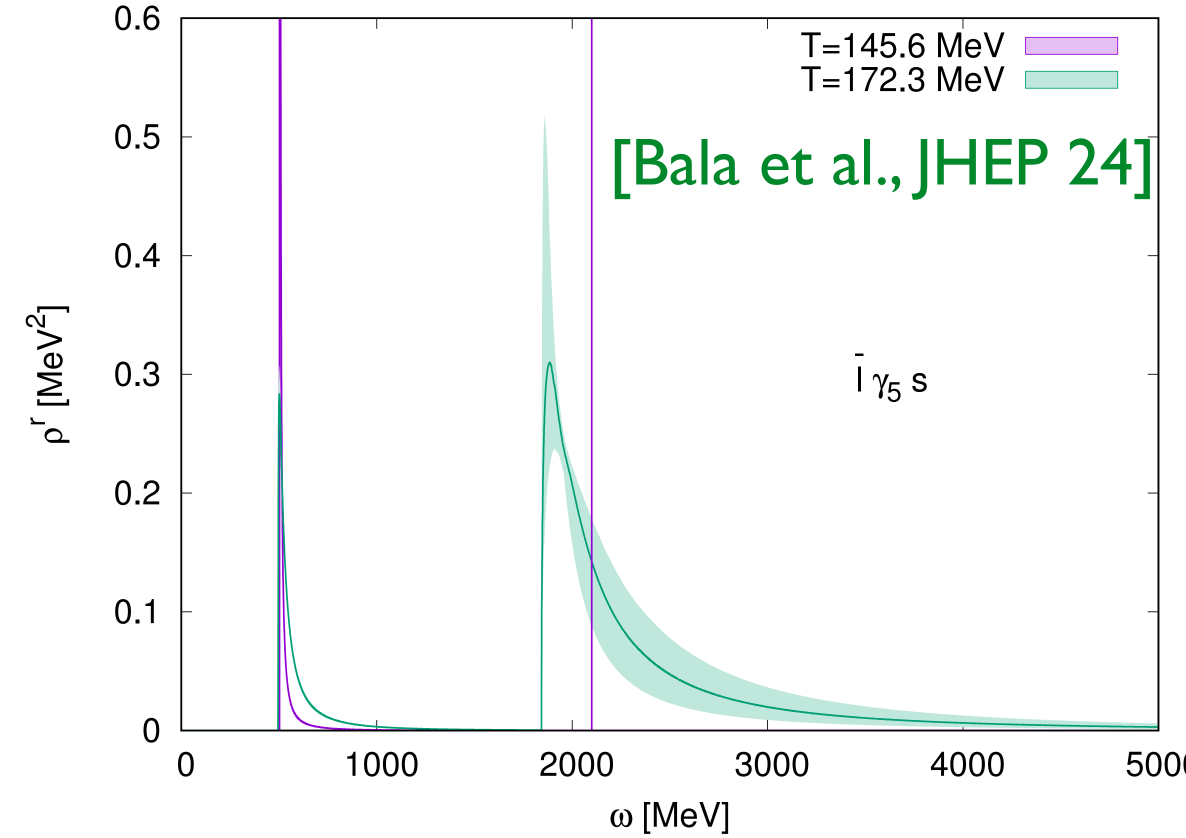


- Works even better than in QCD: simpler spectrum!
- Works better than thermal perturbation theory in a fully perturbative theory!
- General validity of thermoparticle picture, independent of theory!
- Physics: scattering states/Landau damping suppressed for $\frac{T}{m} \sim O(1)$

Does QCD deconfine across the chiral crossover ?

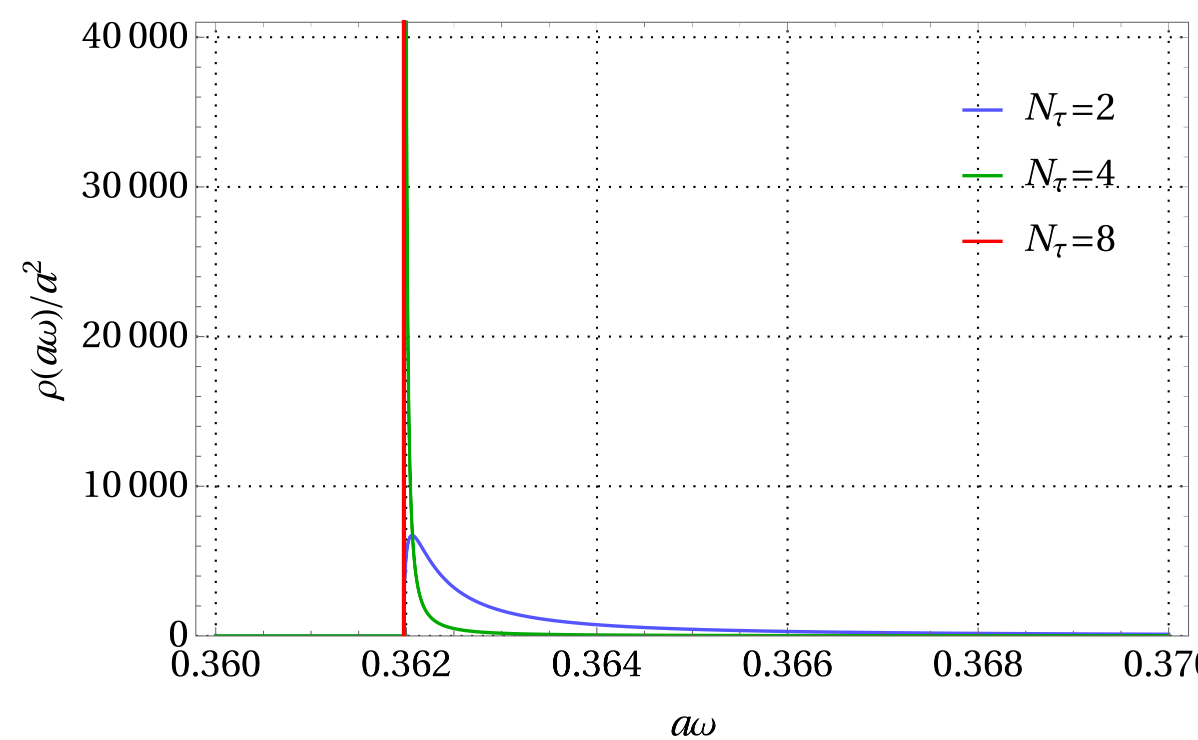
Kaon + Kaon* in full QCD

slightly below and above chiral crossover



Scalar point particle in ϕ^4

no phase transition, no "melting", only "collisional broadening"



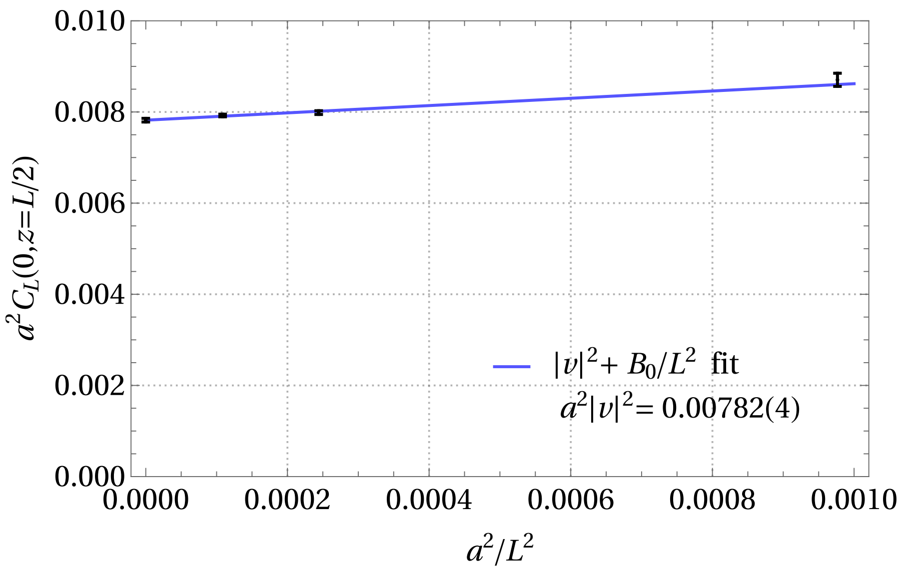
Special case m=0: Goldstone bosons

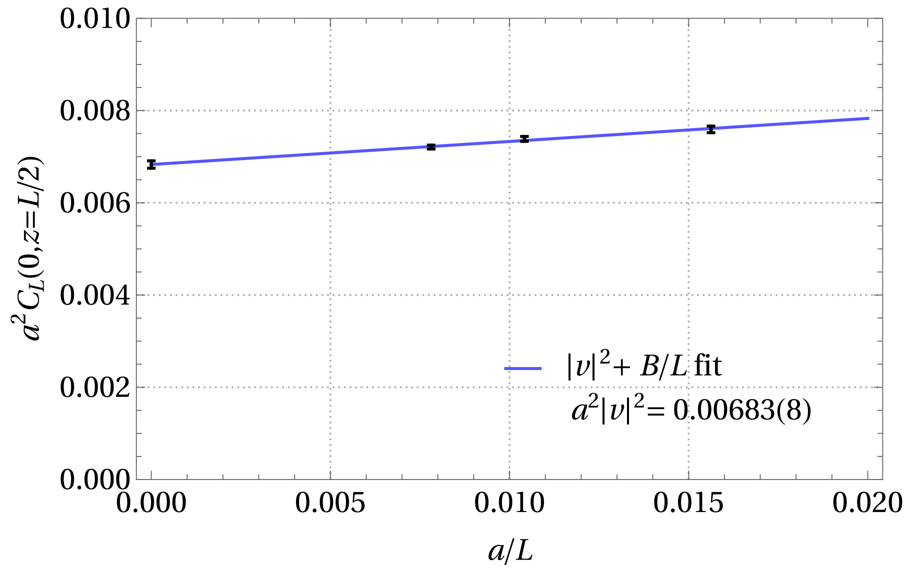
[Bros, Buchholz, PRD 98]

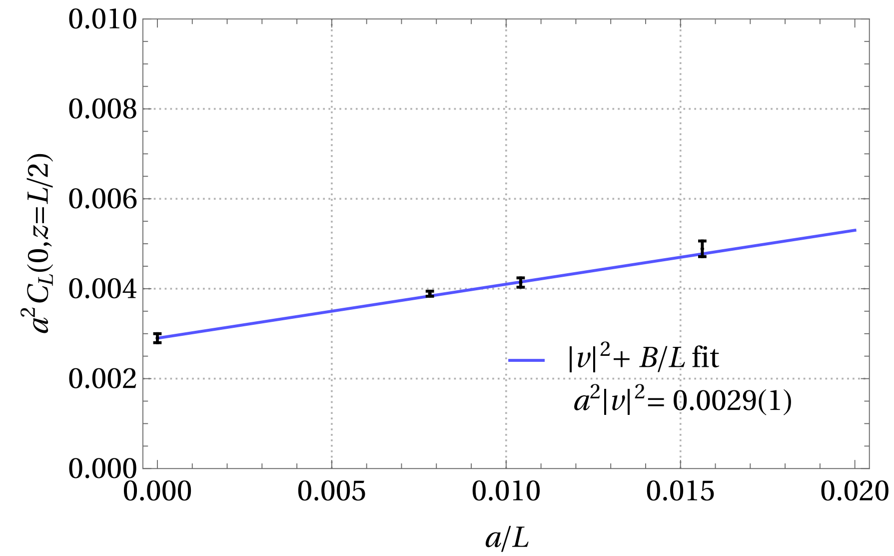
Lattice test in complex U(1) ϕ^4 theory

[Lowdon, O.P. , JHEP 25]

I. Broken phase



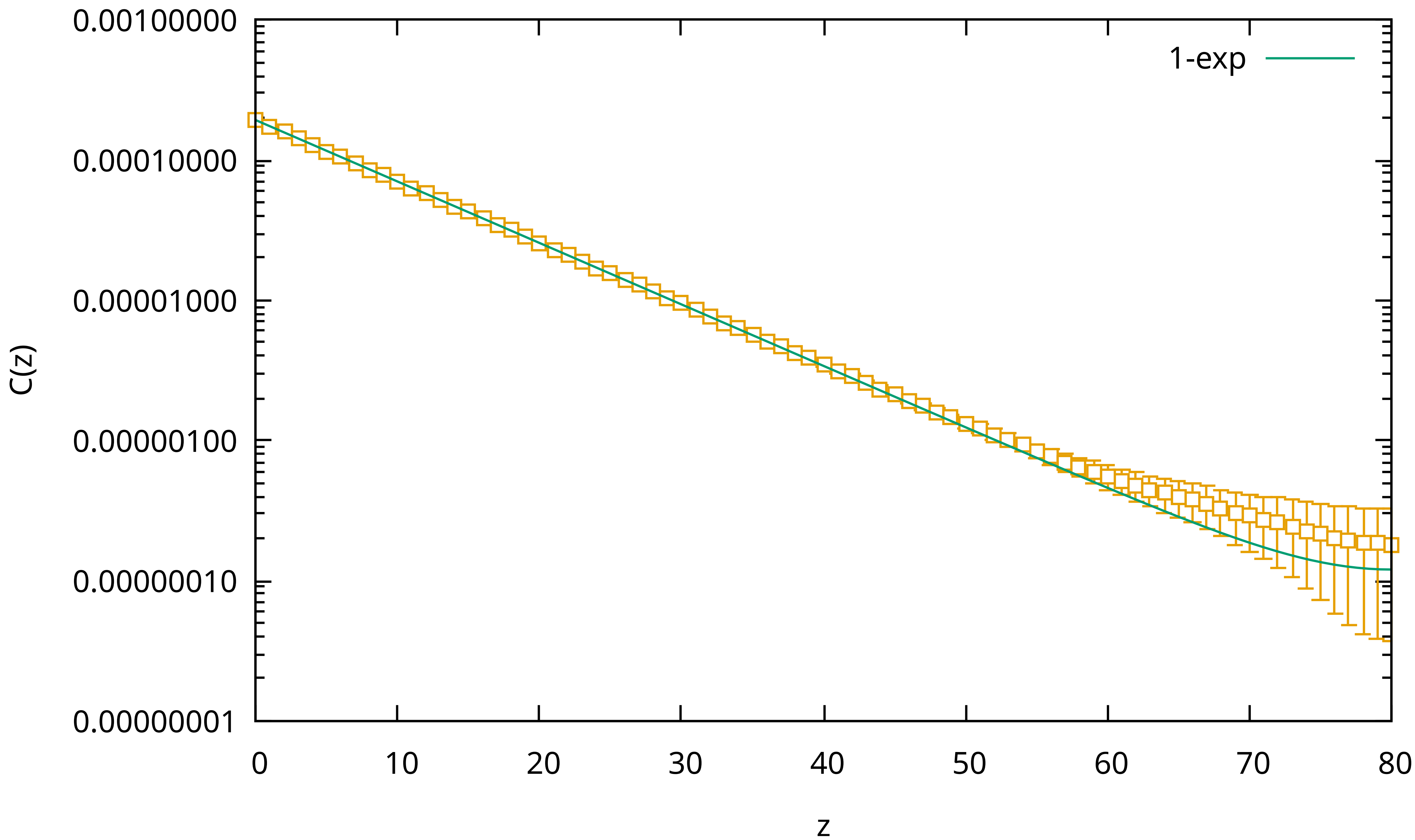




$$C^G(0,\vec{x}) = \frac{\coth\left(\frac{\pi|\vec{x}|}{\beta}\right)}{4\pi\beta|\vec{x}|}\alpha\, e^{-\gamma|\vec{x}|} \xrightarrow{T\to 0} \frac{\alpha_0}{4\pi^2|\vec{x}|^2}$$

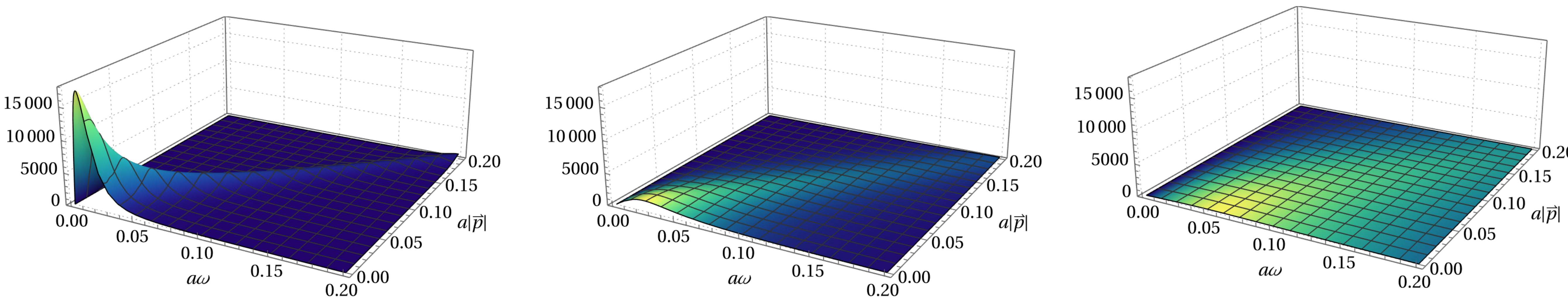
$$\Rightarrow \quad \rho_G(\omega,\vec{p}) = \frac{4\alpha\,\omega\gamma}{(\omega^2 - |\vec{p}|^2 - \gamma^2)^2 + 4\omega^2\gamma^2}$$

Accuracy of correlators

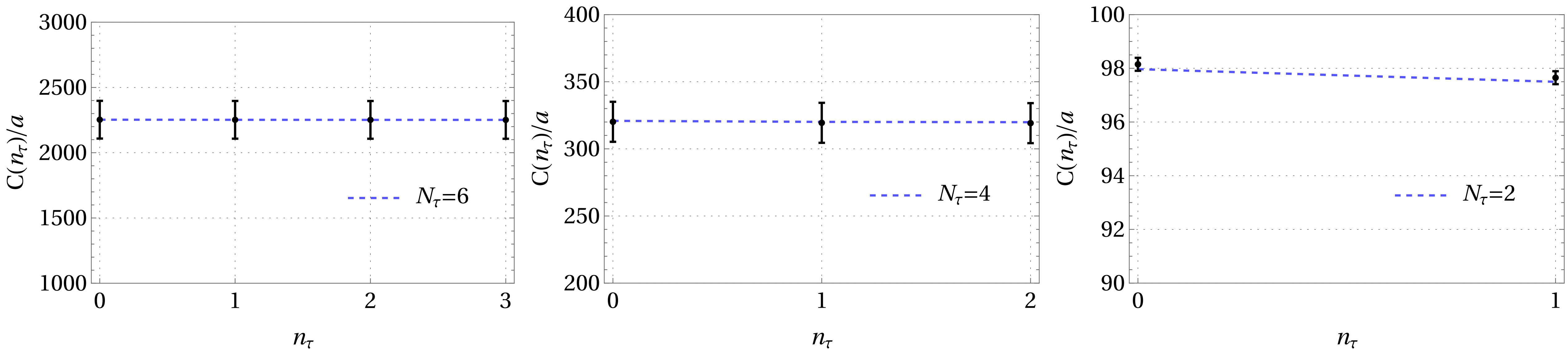


Scalar U(1) $\phi^4, 160^3 \times 2$

II. Symmetric phase

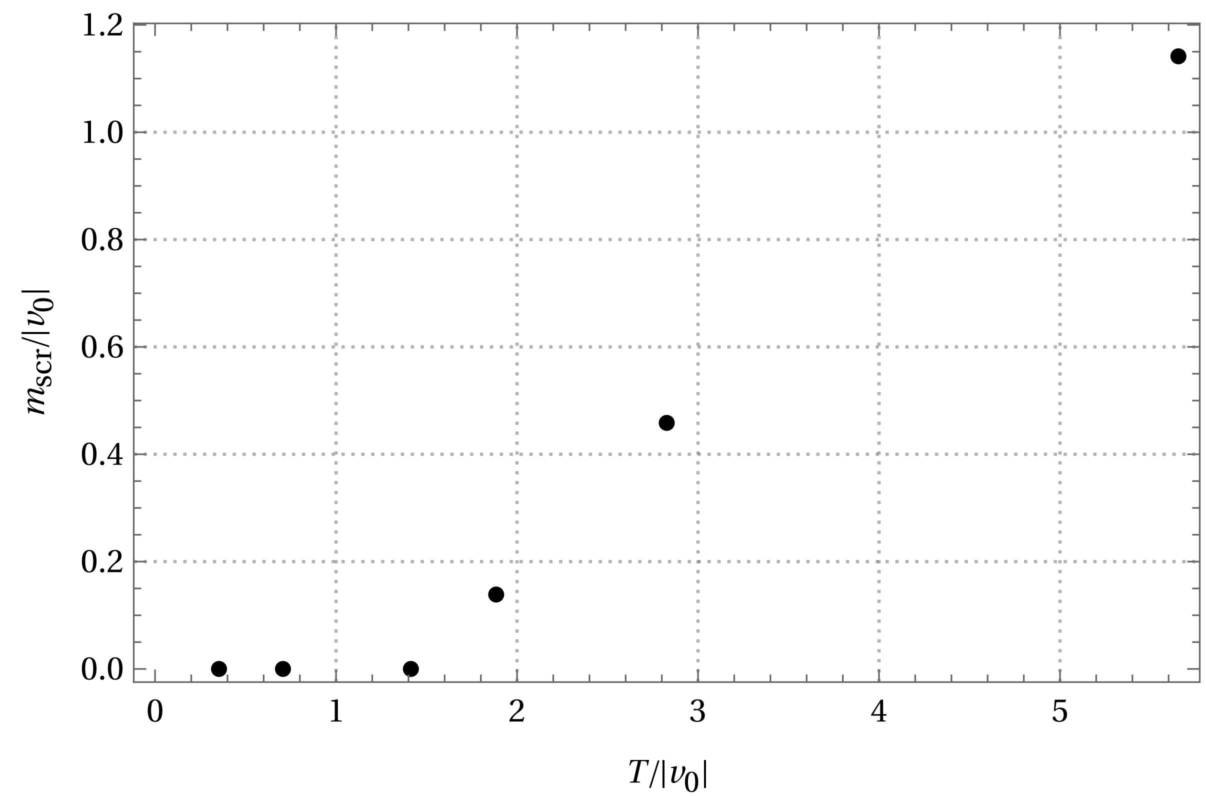


Predict $C(\tau) = \int_0^\infty \frac{d\omega}{2\pi} \frac{\cosh\left[\left(\frac{\beta}{2} - |\tau|\right)\omega\right]}{\sinh\left(\frac{\beta}{2}\omega\right)} \rho_G(\omega, \vec{p}=0)$, compare with simulation



Thermal symmetry restoration by damping

Screening mass of Goldstone boson



$$q = -\lim_{\delta\to 0}\lim_{R\to\infty}\int \frac{d^3\vec{x}}{(2\pi)|\vec{x}|}\; D_\beta^{(+)}(\delta R\,\vec{x})\,\dot{\alpha}(|\vec{x}|) = \lim_{|\vec{x}|\to\infty} D_\beta^{(+)}(\vec{x})$$

Damping determines symmetry restoration

$$C_c(0,\vec{x}) = \frac{\coth\left(\frac{\pi|\vec{x}|}{\beta}\right)}{4\pi\beta|\vec{x}|} D_\beta^G(\vec{x}), \quad D_\beta^G(\vec{x}) \approx \begin{cases} 1 + \ldots & N_\tau = 8, 16, 32 \\ e^{-\gamma|\vec{x}|}, & N_\tau = 2, 4, 6 \end{cases}$$

cold, broken

hot, symm.

Change of dynamics is an effect of the medium → encoded in damping factor!

Conclusions

- Chiral phase transition second order up to the conformal window
- Order of chiral phase transition independent of imag. chemical potential
- Thermoparticles dominate correlators at low T
- Goldstone boson visible on both sides of phase transition
- Symmetry breaking determined by damping of Goldstone mode
  Does $U(1)_A$ matter for QCD?