Effects of $U_A(1)$ and disorder in the Dirac spectrum of finite temperature QCD

Ravi Shanker

The Institute of Mathematical Sciences

May 9, 2026

New Trends in Thermal Phases of QCD, Pan-European University

Reference: arXiv:2602.24227 & Nucl. Phys. B 1025 (2026), H. Pandey, R.S. and S. Sharma

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- Most systems studied so-far are typically at zero or finite temperature with no thermodynamic phase transitions.

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- However the interplay between restoration of different subgroups of chiral symmetry breaking and localization is not very well understood.

Effective restoration of chiral symmetries in QCD

- The non-singlet subgroup of the chiral symmetry i.e. $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$ is effectively restored beyond temperature $T_{pc} = 156.5 \pm 1.5$ MeV [HotQCD collaboration, 2018]

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- We can only discuss its effective restoration at $T_{U(1)} \sim 1.5\ T_{pc} \to$ where $\chi_{top} \sim T^{-8}$ consistent with the dilute instanton gas model prediction

  [P. Petreczky, H-P Schadler, S. Sharma, PLB 2016, S. Borsanyi et. al., Nature 539, 2016]

  or though the degeneracy of meson correlators [G. Aarts et. al. , 2604.1191].

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- Many different intertwined phenomena are happening in the regime $T_{pc} < T < 1.5\ T_{pc}$.

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- Can we learn about thermalization from the properties of Dirac eigenspectrum?

Numerical set-up

- The thermal gauge configurations for $T = 164, 177, 195, 270, 314, 339$ MeV were generated using Möbius domain-wall fermion discretization for 2+1 flavors of dynamical physical quarks and Iwasaki gauge action.

- We calculate first 100-200 eigenvalues of the massless overlap Dirac operator on these thermal gauge configurations

- The chiral crossover transition happens at $T_{pc} \simeq 158$ MeV.
  [M. E. Jaensch, R.S. et. al., Phys. RevD. 111, 034507 (2024)].

- The spatial lattice size in physical units varies between 4-2.5 fm for $T = 164$-339 MeV.

- We use the Wilson flow renormalization scheme following $a \ll \sqrt{8t} \ll 1/T$ to remove the UV divergences while calculating expectation values of the Polyakov loop.

Observables

- We calculate the normalized level ratios,

$$\tilde{r}_n = \min\left(r_n, \frac{1}{r_n}\right), \text{ where } \; r_n = \frac{s_{n+1}}{s_n}$$

  in terms of the spacings $s_n = \lambda_{n+1} - \lambda_n$ between two consecutive Dirac eigenvalues λ_n.

- We study $\langle \tilde{r} \rangle$ in small bins in λ. These observables are free from systematic errors related to unfolding.

- It is well known that universal level-spacing fluctuations of Dirac eigenvalues at $T < T_{pc}$ can be described in terms of random matrices belonging to Gaussian unitary ensemble (GUE).

  [J. J. M. Verbaarschot, Phys. Rev. Lett. 72, 1994]

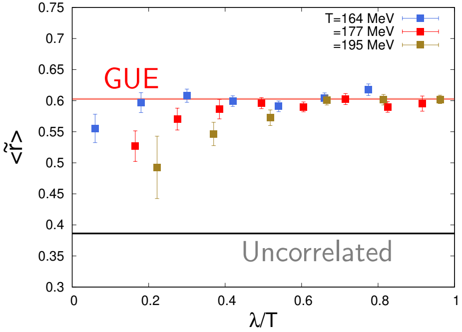
- We will instead study the level ratios at $T > T_{pc}$ and use it to categorize different regions of the Dirac eigenvalue spectrum.

Level ratios of Dirac eigenvalues as for $T \gtrsim T_{pc}$

- In most λ/T bins the values $\langle\tilde{r}\rangle$ are consistent with the GUE prediction $\rightarrow$ bulk eigenmodes.

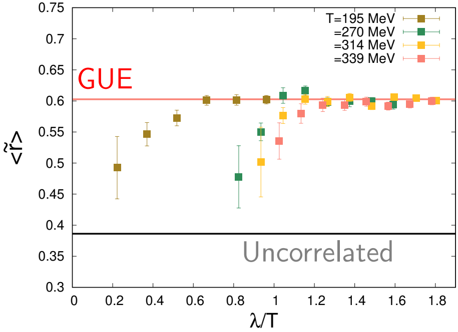
  [Plots from R.S., H. Pandey, S. Sharma, 2602.24227].

- Few eigenmodes appearing in the deep infrared part of the spectrum have level ratio that is intermediate between GUE and uncorrelated ensembles $\rightarrow$ intermediate eigenmodes.



Level ratios of Dirac eigenvalues for $T \gtrsim T_{U(1)}$

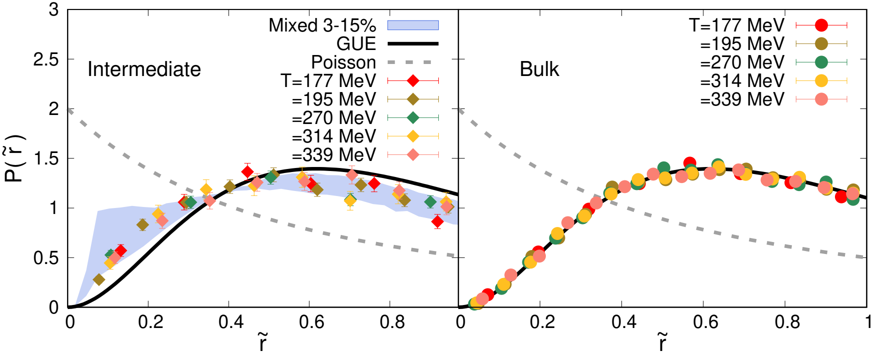
- A gap in the infrared appears at $T_{U_A(1)} \sim 1.5\,T_c$.
- Intermediate eigenvalues still persist beyond the gap but become more rarer with increasing T.



A random matrix model for the intermediate eigenmodes

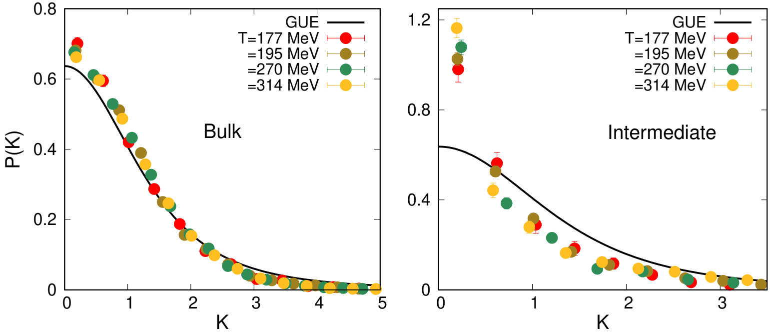
- For the intermediate eigenmodes we proposed a matrix model where a large matrix belonging to GUE has a 3-15% admixture of eigenvalues with Poisson statistics.

[R.S., H. Pandey, S. Sharma, 2602.24227]

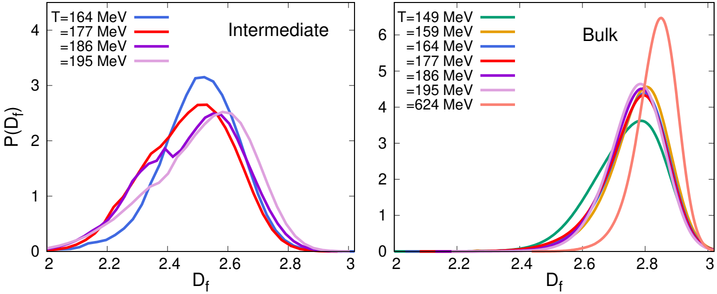


Structural features of the Dirac eigenfunctions

- We next monitor how Dirac eigenstates change due to a twist applied on one of the spatial boundaries $\psi(x+L) = \mathrm{e}^{\mathrm{i}\eta}\psi(\mathrm{x})$.
- Curvature $K_i = \lim_{\eta\to 0}\left|\frac{\partial^2 \lambda_i}{\partial \eta^2}\right|$. for the bulk and intermediate eigenmodes are very different.



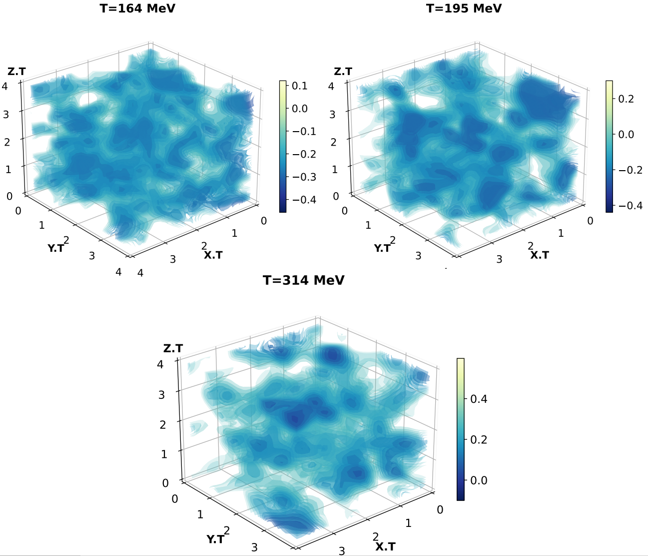
Intermediate eigenmodes at $T \gtrsim T_{pc}$ and criticality



- The fractal dimensions of intermediate modes $D_f \sim 2.5$. Using the fact that $D_f = 3 - \frac{\beta}{\nu}$ [K. Jansen and C. B. Lang, Phys. Rev. Lett. 66, 1991], we can interpret the value of D_f using β and ν, the critical exponents of $O(4)$ model.

- Indicates that $U_A(1)$ is not effectively restored!

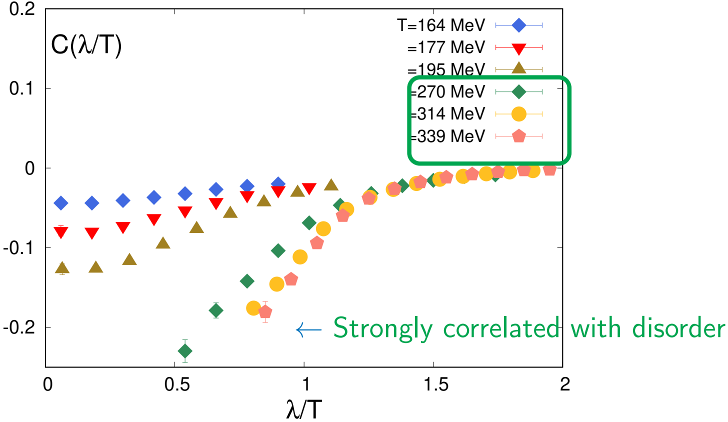
Disorder in the gauge fields quantified in terms of Polyakov loop fluctuations



Change in localization properties of intermediate IR eigenmodes due to disorder

- The correlation of Dirac eigenmodes with the local fluctuations in the renormalized Polyakov loop values can be quantified in terms of

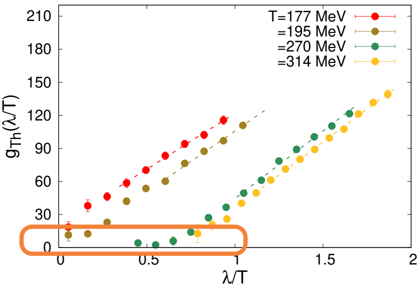
$$C_n = \sum_{\mathbf{x}} |\Psi_n(\mathbf{x})|^2 \left[\mathrm{Re}.P(\mathbf{x}) - \langle \mathrm{Re}.P(\mathbf{x})\rangle\right].$$

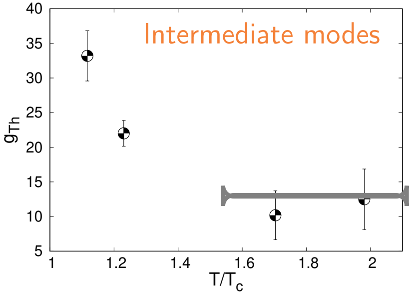


Thouless conductance as an effective order parameter for $U_A(1)$?

- We calculate the Thouless conductance which measures structural rigidity of eigenvectors

$$g_{\mathrm{Th}} = \frac{1}{\langle s_i \rangle} \langle K_i \rangle_{\eta \to 0} \quad \text{where} \quad K_i = \left| \frac{\partial^2 \lambda_i}{\partial \eta^2} \right| .$$



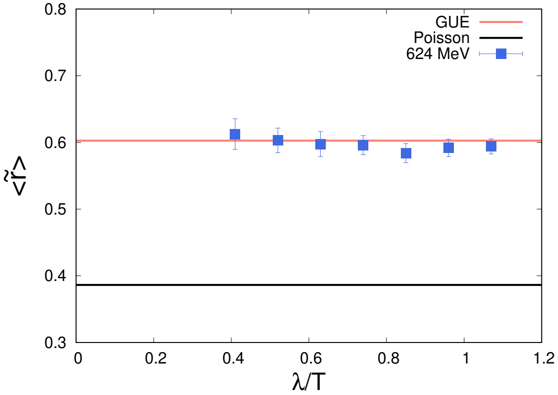


What more can the Dirac spectrum tell us: thermalization

- Thermalization in an isolated classical system is closely related to the ideas of chaos and ergodicity [See for e.g Review by J. M. Deutsch, 2018].
- Chaos defined in terms of exponential separation between two initial close trajectories in phase space is no longer valid for quantum systems.
- The first major attempt to understand quantum analogs of classical chaos was by Bohigas, Giannoni, and Schmit (BGS, 1984).
- They conjectured that spectral fluctuations of quantum systems whose classical dynamics are chaotic follow the predictions from one of the three classes of random matrix theory (RMT).

A demonstration of the BGS conjecture

- At $T \sim 600$ MeV all infrared Dirac eigenvalues in quenched QCD below the magnetic scale $g^2 T/\pi$ has level spacing ratios consistent with GUE.



A demonstration of the BGS conjecture in QCD

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- We next calculate the gauge-invariant distance measure

$$D(U_l, U_l^{'}, t) = \frac{1}{N_P} \sum_P \frac{1}{N_c} \left| \mathrm{tr} U_P - \mathrm{tr} U_P^{'} \right| ,$$ [B. Müller & A. Traynov, 1992]

defined in terms of plaquettes U_P, starting from two initial states that have infinitesimally close initial occupation numbers.

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- We could extract a positive Lyapunov exponent for different choices of initial occupation numbers $\rightarrow$ characteristic of a chaotic system. This is a non-trivial realization of the BGS conjecture for low-momentum gluons.

[H. Pandey, R. Shanker, S. Sharma, Nucl. Phys. B 1025, 2026].

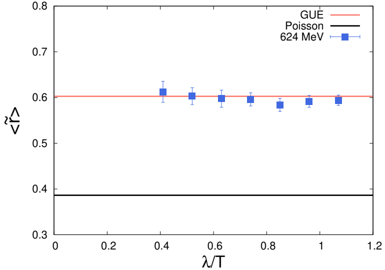
Summary and Outlook

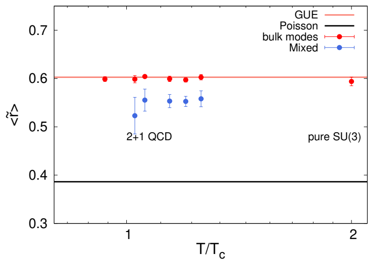
- Appearance of IR Dirac eigenmodes with intermediate level statistics has a different physical origin depending on whether or not the $U_A(1)$ is effectively restored.
- These intermediate eigenstates at $T \gtrsim T_{pc}$ carry information about the criticality of the phase transition $\rightarrow$ likely to be $O(4)$.
- We have observed a drop of Thouless conductance and saturating at $T > T_{U_A(1)} \rightarrow$ acts as an effective order parameter for $U_A(1)$ breaking.
- At $T > T_{U_A(1)}$ these intermediate IR eigenmodes are correlated with random disorder due to wells in the local Polyakov loop values.
- We also discussed a non-trivial realization of the BGS conjecture in quenched QCD at $T \sim 600$ MeV. Has implications for understanding thermalization of strongly interacting low-momentum gluons.

Thank you!

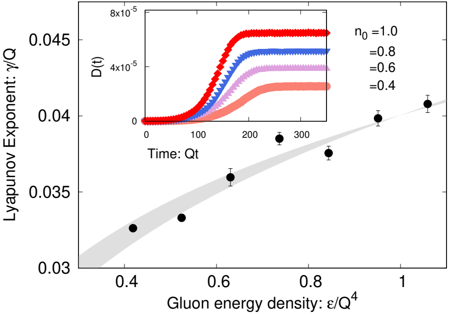
- $\lambda_0,\ \lambda_1, ...,\ \lambda_{n-1},\ \lambda_n,\ \lambda_{n'},\ \lambda_{n+1},\ \lambda_{n+2}, ...,\ \lambda_{N-1}$
- Here λ_i, where $i = 0, .., N-1$ are eigenvalues of our original GUE-block of size N. The three new level ratios that arise as a result are $r_{n_1} = \frac{\lambda_{n'}-\lambda_n}{\lambda_n-\lambda_{n-1}}$, $r_{n_2} = \frac{\lambda_{n+1}-\lambda_{n'}}{\lambda_{n'}-\lambda_n}$ and $r_{n_3} = \frac{\lambda_{n+2}-\lambda_{n+1}}{\lambda_{n+1}-\lambda_{n'}}$, which are normalized to give us $\tilde{r}_{n_i} = \min\left(r_{n_i}, \frac{1}{r_{n_i}}\right)$, $i = 1, 2, 3$.
- The new eigenvalues are accepted only if all three locally induced $\tilde{r}$ are compatible with the target distribution.

Backup: Level ratios of intermediate and bulk eigenmodes versus T

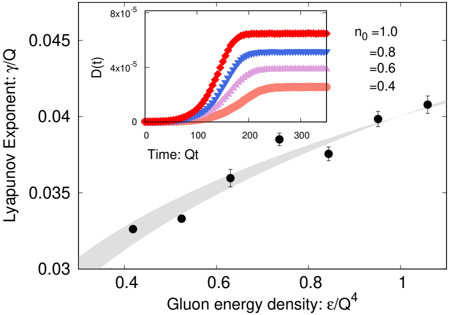




Chaotic nature of a classical non-thermal state of QCD



- The gauge links $U_{i,x}$ and the electric fields $E^i_{a,x}$ are evolved in time according to classical Hamiltonian equations of motion.
- Rapid memory loss $\rightarrow$ enters a self-similar scaling regime.
- Gluon distribution function exhibits a scaling relation of the form, $g^2 f_g(|\mathrm{p}|, t) = (Qt)^{-\frac{4}{7}}\ f_s\left[(Qt)^{-\frac{1}{7}} \frac{|\mathrm{p}|}{Q}\right]$.



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between two close gauge-trajectories which at initial time $t = 0$ are characterized by n_0 and $n_0 + \Delta n_0, \ \Delta n_0 = 0.001$.

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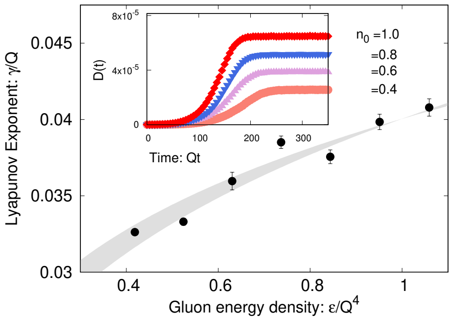

- System exhibits chaotic behavior resulting in $D(t)$ increasing exponentially with time.
- The positive Lyapunov exponent γ is characteristic of a chaotic system.