The far infrared mobility edge of the QCD Dirac spectrum

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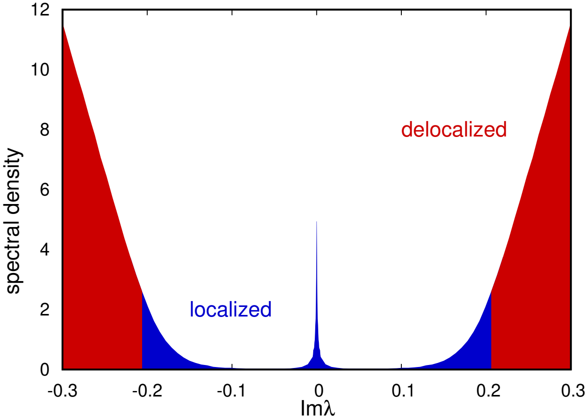
Spectral density of the lattice overlap Dirac operator for $T > T_c$



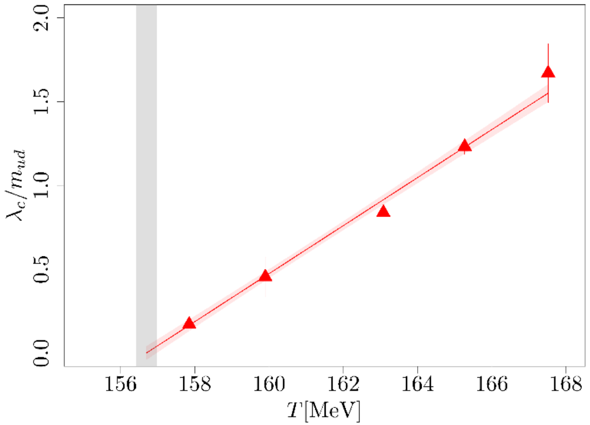
mobility edge at $\pm\lambda_c$

Peak at zero in the spectral density!

Edwards et al. PRD 61 (2000); Alexandru & Horvath, PRD 92 (2015); 2404.12298; Kaczmarek, Mazur, Sharma, PRD 104 (2021) 2021

Mobility edge vs. temperature

$N_f = 2+1$ lattice with physical staggered quarks

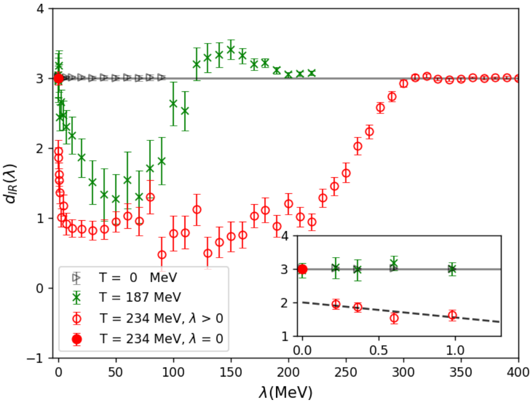


Giordano, TGK, Pittler, arXiv:2602.10921.

Are all the $|\lambda| < \lambda_c$ modes localized?

effective dimension across the spectrum

- Modes close to zero get delocalized!
- Is there another mobility edge?
- How to focus on $\lambda \to 0$?
- Need large volumes.



Meng, Sun, Alexandru, Horváth, Liu, Wang, Yang, JHEP 12 (2024) 101.

Instanton-based random matrix model of the spectral peak

- (Anti)instanton
  $\rightarrow$ zero eigenvalue of $D(A)$ with $(-)+$ chirality eigenmode

- High T:
  large instantons "squeezed out" in the temporal direction
  $\rightarrow$ dilute gas of instantons and antiinstantons

- Zero modes exponentially localized:

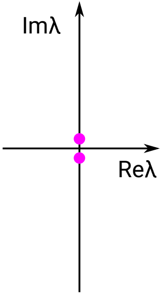
$$\psi(r) \propto e^{-\pi T r}$$

Instanton-antiinstanton pair

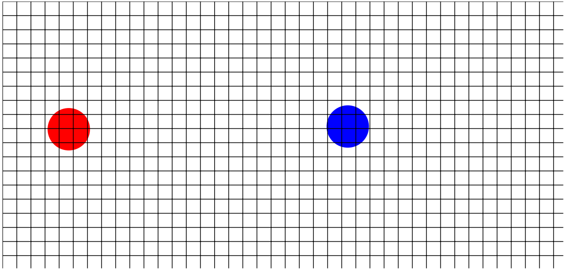
The Dirac operator in the subspace of zero modes

$$D(A) = \begin{pmatrix} 0 & iw \\ iw & 0 \end{pmatrix} \qquad w \propto e^{-\pi Tr}$$

Spectrum of $D(A)$



Instanton and antiinstanton



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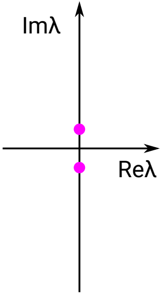

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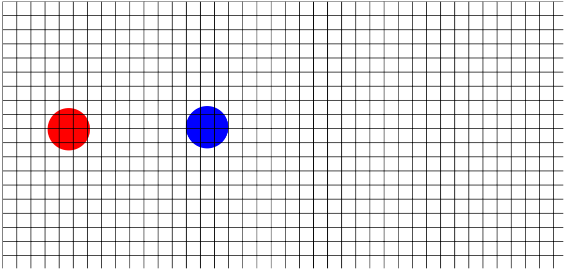

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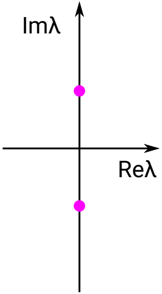

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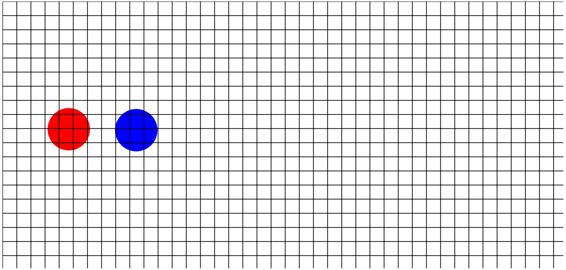

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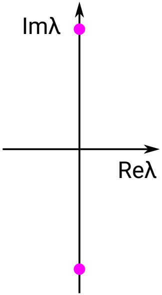

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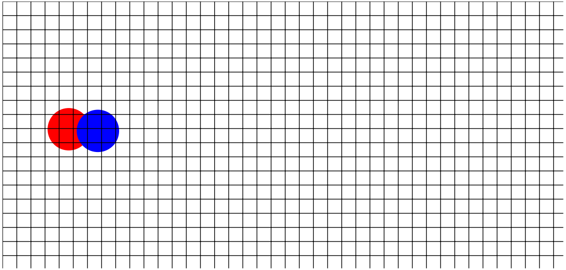
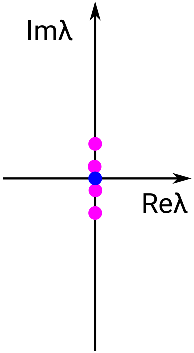
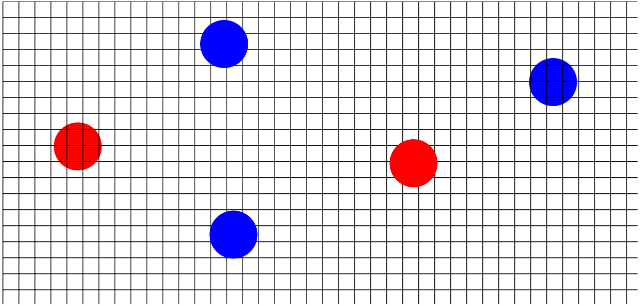


Spectrum of $D(A)$ in dilute gas of instantons

The Dirac operator in the subspace of zero modes

Spectrum of $D(A)$ Instantons and antiinstantons





n_i instantons n_a antiinstantons

$\rightarrow$ $|n_i - n_a|$ exact zero modes + mixing near zero modes

Dirac operator in the subspace of zero modes (ZMZ)

Work by E.V. Shuryak, J.J.M. Verbaarschot, T. Schäfer (1990-2000)...

- Given n_i instantons, n_a antiinstantons in 3d box of size L^3
- Construct $(n_i + n_a) \times (n_i + n_a)$ matrix:

$$D = \left(\begin{array}{c|c} \overbrace{\quad 0 \quad}^{n_i} & \overbrace{\quad iW \quad}^{n_a} \\ \hline iW^\dagger & 0 \end{array} \right)$$

- $w_{ij} = A \cdot \exp(-\pi T \cdot r_{ij})$ $\qquad$ r_{ij} is the distance of instanton i and antiinstanton j

Random matrix model of $D(A)$ in the zero mode zone

- How to choose instanton numbers (n_i, n_a) and locations?

- Quenched lattice $T > 1.05\,T_c \quad \rightarrow \quad$ free instanton gas

  Bonati et al. PRL 110 (2013); Vig R. & TGK, PRD 103 (2021)

  - n_i and n_a independent identical Poisson-distributed

$$p(n_i, n_a) = e^{-\chi V} \cdot \frac{(\chi V/2)^{n_i}}{n_i!} \cdot \frac{(\chi V/2)^{n_a}}{n_a!}$$

    χ is the topological susceptibility

  - Locations random (uniform)

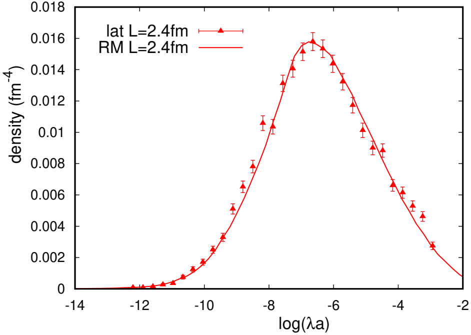
- $\rightarrow$ D(A) in quenched QCD $\Leftrightarrow$ ensemble of random matrices

Fit parameters to quenched lattice Dirac spectrum

$T = 1.1\,T_c$ overlap Dirac spectrum

- Two parameters:
  - χ – topological susceptibility: from exact zero modes $\rightarrow \chi = \langle Q^2 \rangle / V$
  - A – prefactor of the exponential mixing between zero modes
- Fit A to distribution of Dirac eigenvalues (lowest eigenvalue)

$L = 2.4$fm fit



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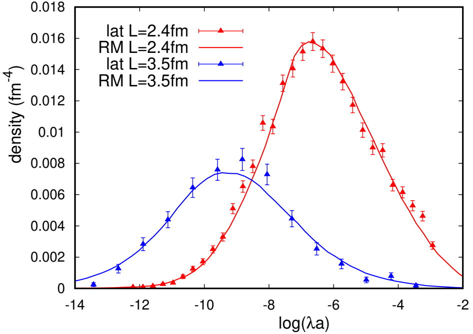
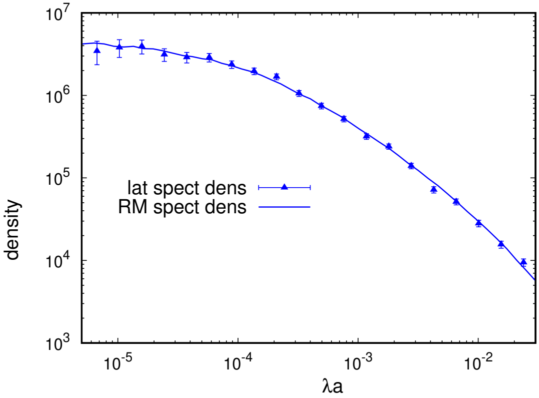
$L = 2.4$fm fit

$L = 3.5$fm prediction



Spectral density

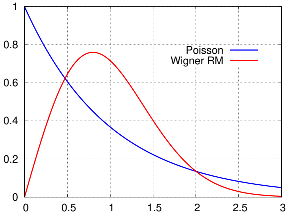
Lattice vs. random matrix model, $L = 3.5$ fm

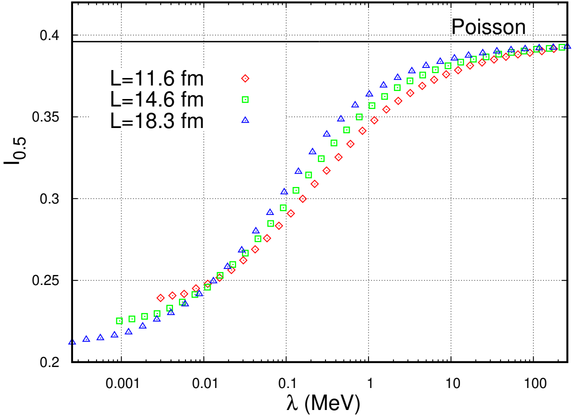


Delocalization in the far infrared

$\chi_{top} \approx \chi_{top}(T_c)$ in quenched

$$I_{0.5} = \int_0^{0.5} \rho_{unf}(s) ds$$

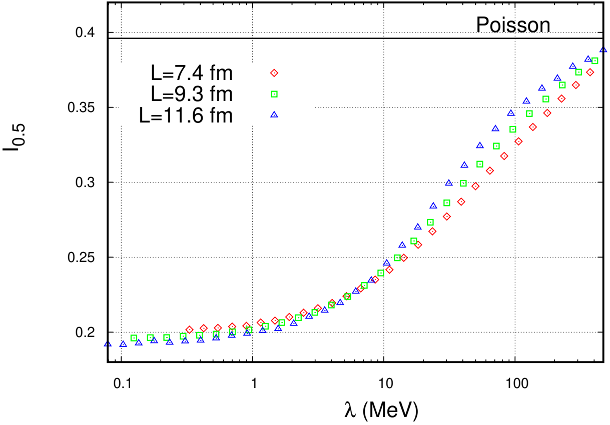




Delocalization in the far infrared

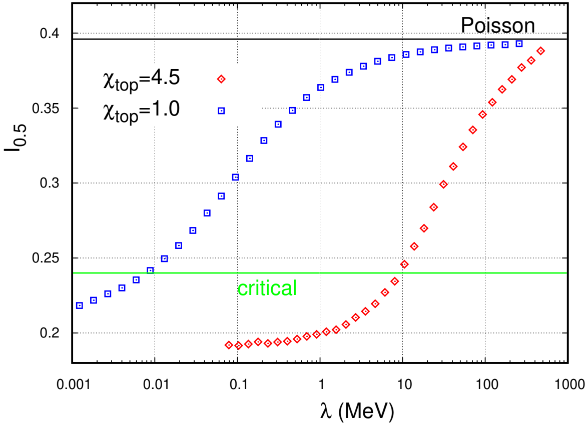
$\chi_{top} \approx 4\chi_{top}(T_c)$ in quenched

$$I_{0.5} = \int_0^{0.5} \rho_{unf}(s) ds$$



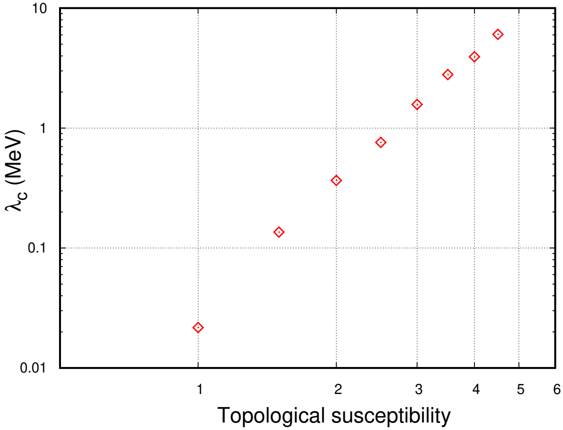
Delocalization for different instanton densities

λ_c decreses with χ_{top}



The mobility edge versus χ_{top} in units of $\chi_{top}(T_c)$

- $\lambda_c \to 0$ as $\chi_{top} \to 0$
- χ_{top} controlled by T, m_q



Conclusions

- The QCD Dirac spectrum above T_c has a far infrared mobility edge
- $\lambda_{cIR} \to 0 \quad \text{as} \quad m_q \to 0 \quad \text{and/or} \quad T \to \infty$
- Consistent with exact constraints on the Dirac spectrum (see Matteo's talk)
- Spectral peak singular at zero, but power has nontrivial corrections
- The far infrared part of the spectrum drives $U(1)_A$ breaking above T_c.