The IR Phase: Firm, Loose and Open Ends

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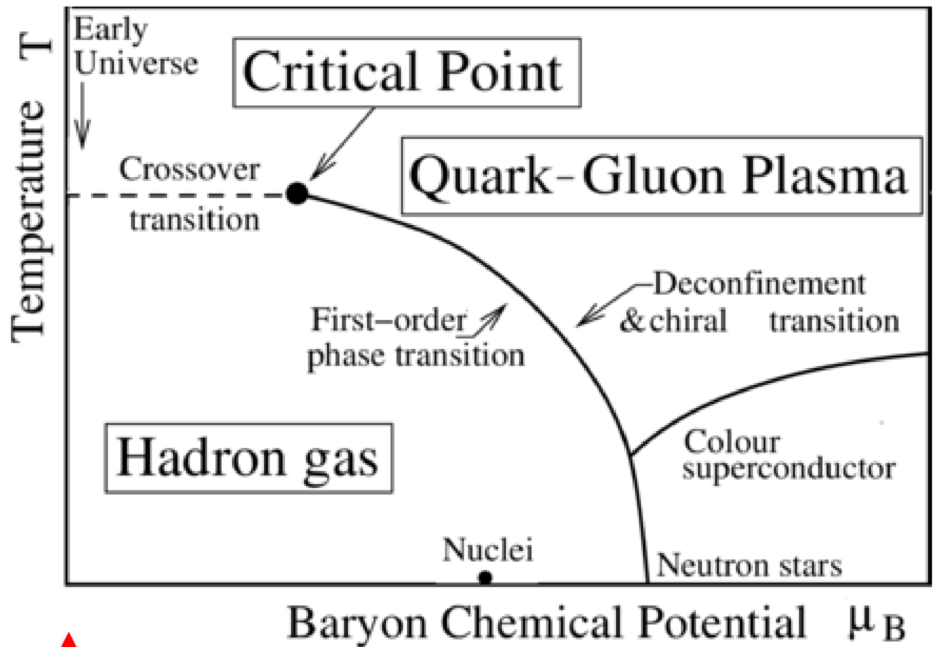
Literature: [with degrees of separation]

0-th	0-th	1-st	1-st	2-nd
1906.08047	2305.09459	1405.2968	hep-lat/0607031	1807.03995
1502.07732	2310.03621	1412.1777	hep-lat/0610121	1809.07249
2103.05607	2404.12298		hep-lat/0703010	2110.11266
2110.04833			0803.2744	2205.11520
2509.03509				2207.13569
				2212.09806
basics	newer evidence (thermal)	evidence (large N_f)	$< F^2 >$ & Dirac spectrum	math tools

Conventional View of the Territory: Phases of Strongly-Interacting Matter

$T_c \approx 155$ MeV
chiral crossover



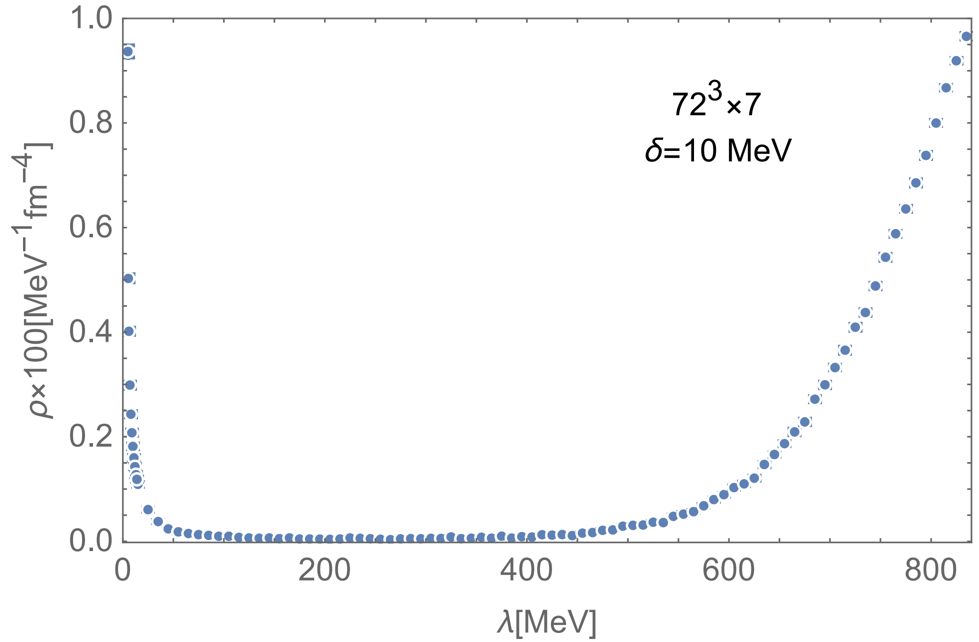
LATTICE

MOSTLY SPECULATION

Q: IS THERE SOMETHING QUALITATIVELY NEW LEFT TO DISCOVER IN QCD AT $\mu_B = 0$?

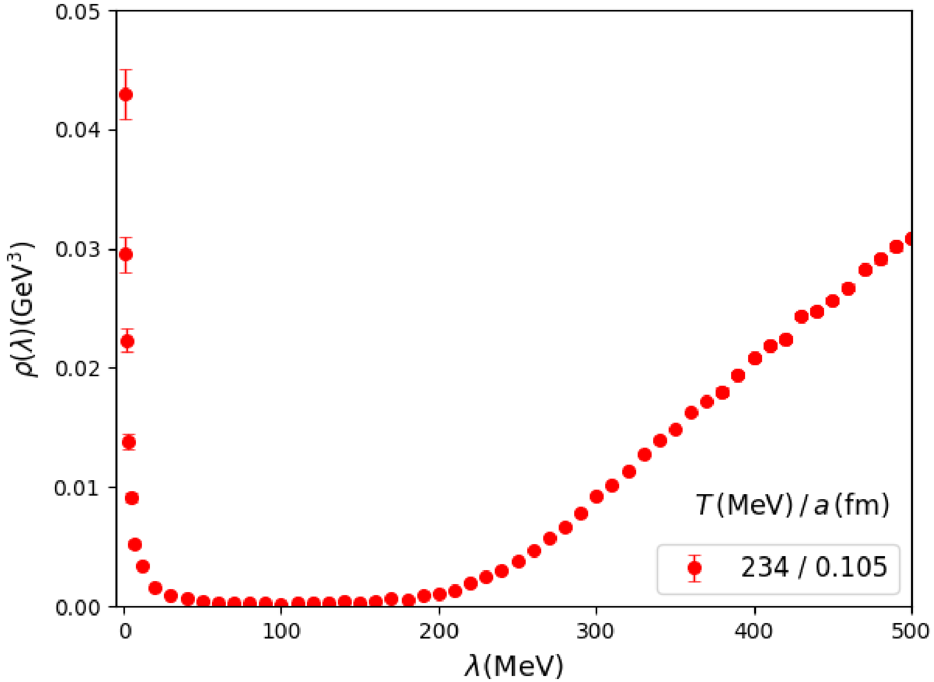
This is the mantra of PBP meetings for me…

Reasons for “new” have to do with things like this:



AA & IH unpublished

N_f=0
a=0.085 fm
T=1.12 T_c



2305.09459

N_f=2+1 (real world)
L=5.0 fm
Clover dynamical q-s

Two important steps

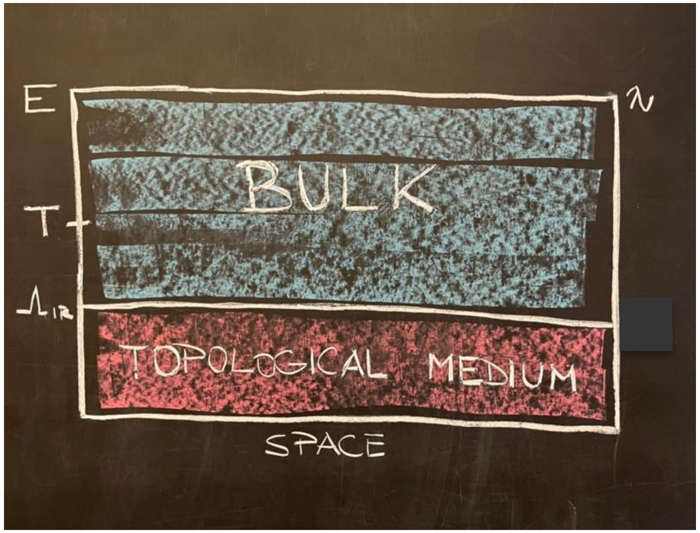
1) 1502.07732

- bimodality is almost perfect
- universal for vectorlike SU(3) gauge theories
- it is neither UV nor IR artifact
- signifies a <u>phase</u> because DOFs in IR mode are different (deconfined)

Phase transition based on:

- **SINGLE-COMPONENT system ----> MULTI-COMPONENT**

bimodality --- IR-UV separation --- IR-Bulk separation --- IR-Bulk decoupling



Origin of this representation:

BULK component

IR component

Two important steps…

2) 1906.08047

- Near-pure negative power law accumulation of Dirac modes in IR component

$$\rho(\lambda) \propto \lambda^{-1+\delta} \ , \ \ \lambda \to 0 \ , \ \ \delta \text{ small}$$

$\delta << 1$ at least near thermal transitions in QCD-like cases

- **SCALE INVARIANCE OF GLUE IN IR COMPONENT**

- Classification of phases NOT BASED ON SYMMETRIES but also YES

$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \rho(\lambda) \propto \lambda^p \ , \ \ \lambda \to 0$$

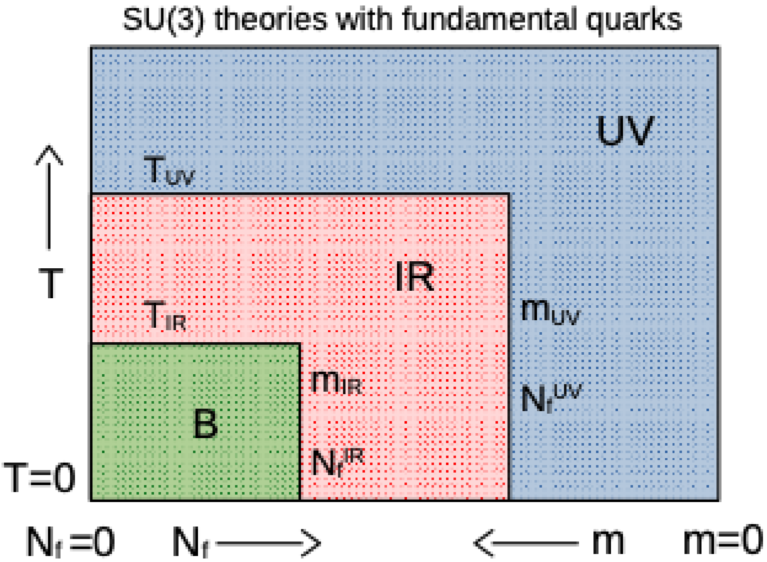
B = single-component & IR-broken

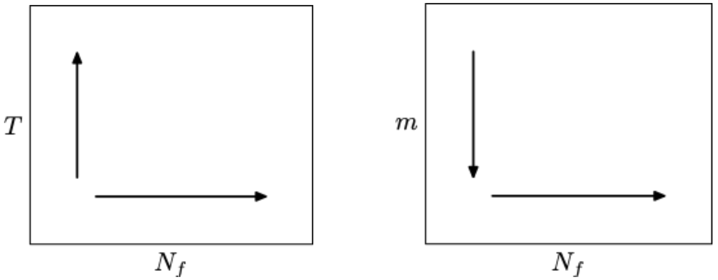
IR = multi-component & IR-symmetric

UV = single-component & IR-trivial

Two important steps…

2) 1906.08047 …led to things like this





Changes in indicated directions can induce transitions B→IR or IR→UV. Also 1502.07732

Two important steps...

2) 1906.08047 ...led to things like this [IR phase of thermal QCD]

HADRON GAS	NEAR-PERFECT FLUID	WEAKLY-COUPLED QGP
IR-broken	IR-symmetric	IR-trivial

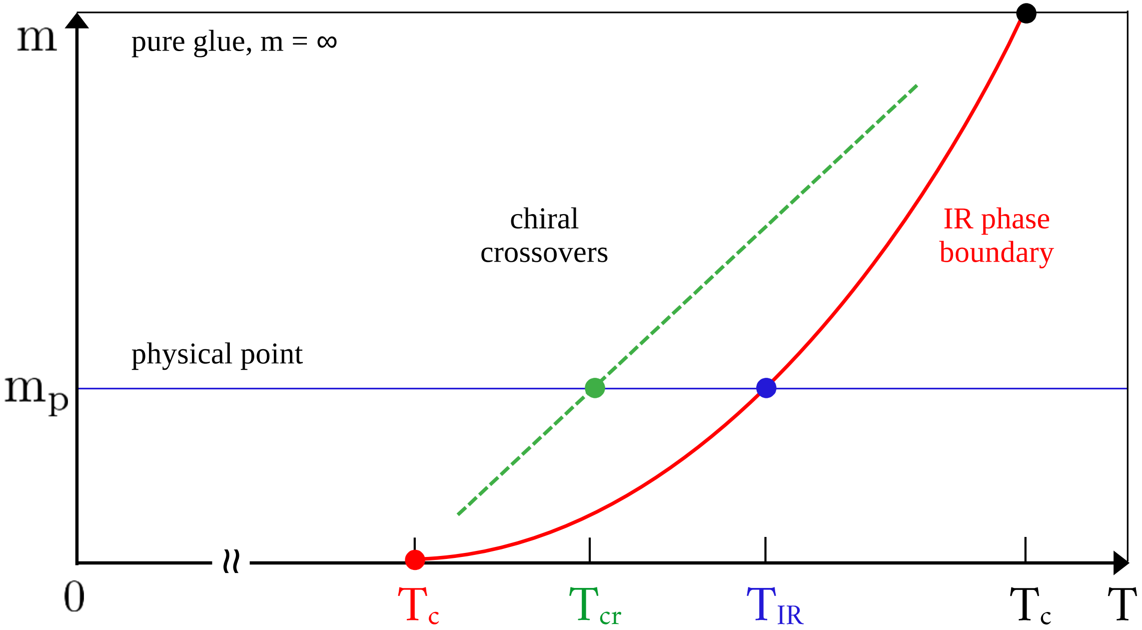
STRONGLY COUPLED	STRONGLY COUPLED	PERTURBATIVE
p = 0	p □ -1	p > 0

$$T_c \quad < \quad T_{IR} \quad < \quad T \quad < \quad T_{UV}$$

≈155 MeV   200-230 MeV   perturb

Two important steps…

2) 1906.08047 …led to things like this [IR phase of thermal QCD]

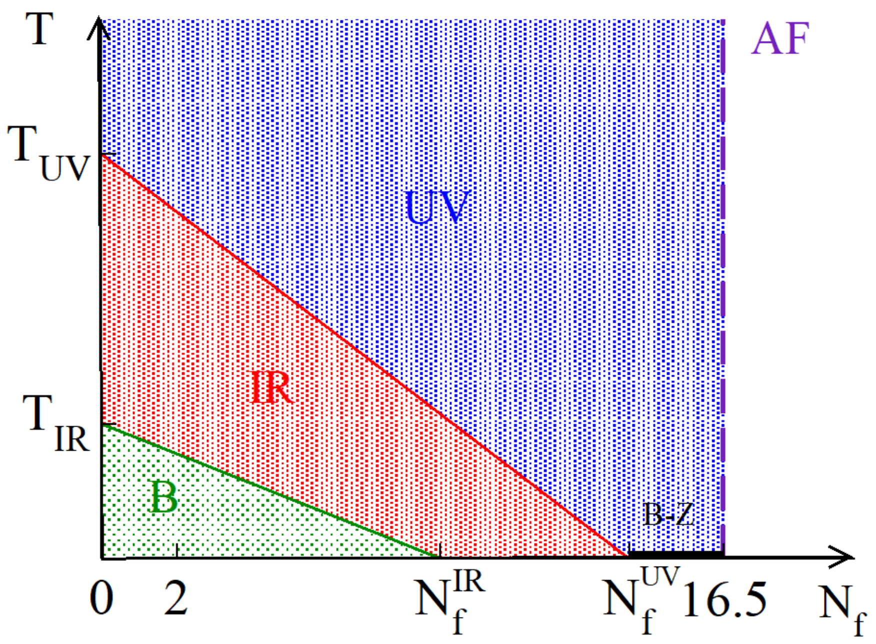


Two important steps…

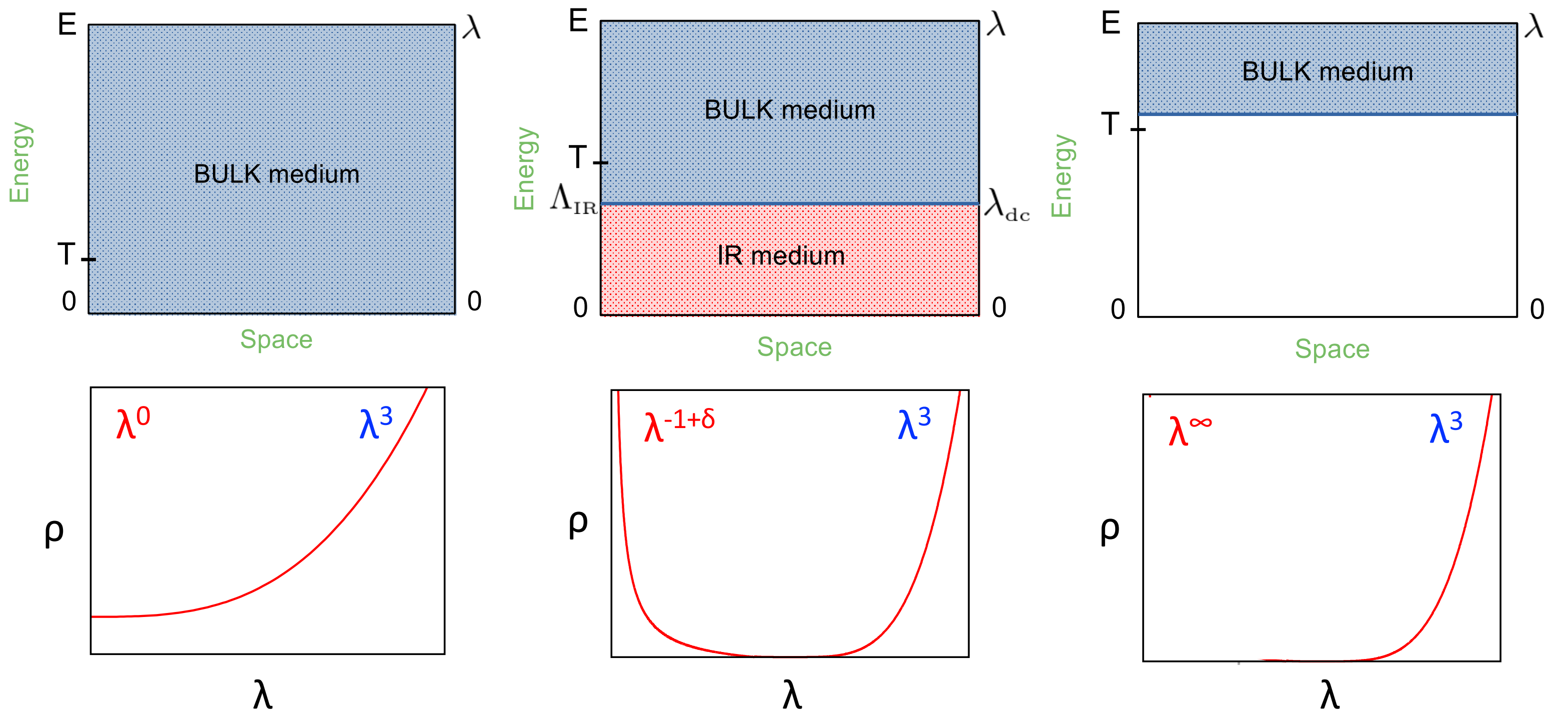
2) 1906.08047 …led to things like this

Important B→IR light-flavor case (T=0):
1405.2968, 1412.1777, 1906.08047

IR PHASE = STRONGLY COUPLED PART OF CONFORMAL WINDOW



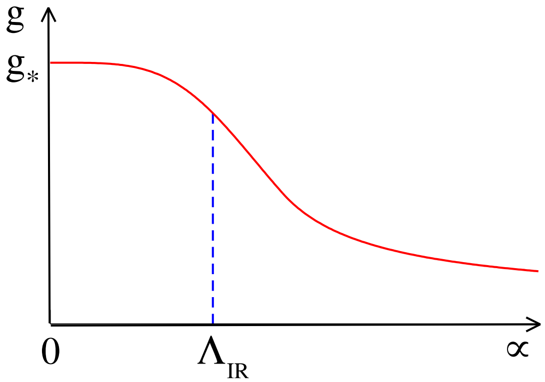
Two important steps...put together

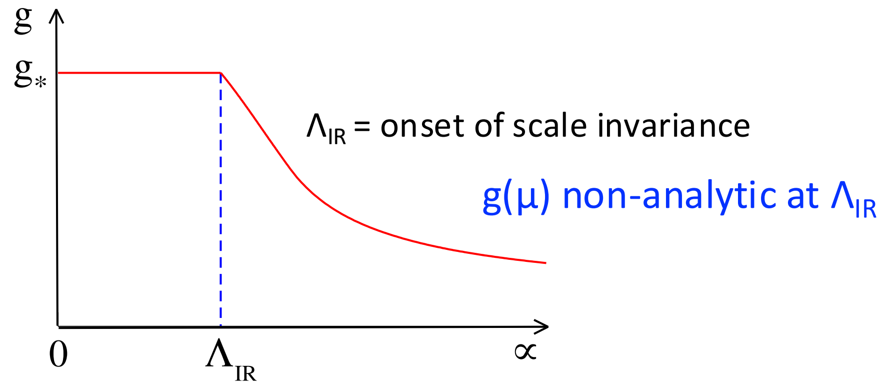


- IR TRANSITION AT 200 MeV < T_{IR} < 230 MeV <u>A PHASE TRANSITION</u>
- (1) IR BECOMES AN AUTONOMOUS COMPONENT: <u>IR-BULK DECOUPLING</u>
- (2) GLUE OF IR COMPONENT BECOMES <u>SCALE INVARIANT</u> (AT LEAST ASYMPTOTICALLY)

How did the connection to Anderson-like localization arise? 1906.08047, 2103.05607, 2110.04833

Came from early considerations concerning IR scale invariance: asymptotic or exact?





Q: If exact, how does the non-analyticity arise?

A: IR-Bulk decoupling ---> IR glue fluctuates independently of Bulk glue
---> mismatch/non-analyticity natural

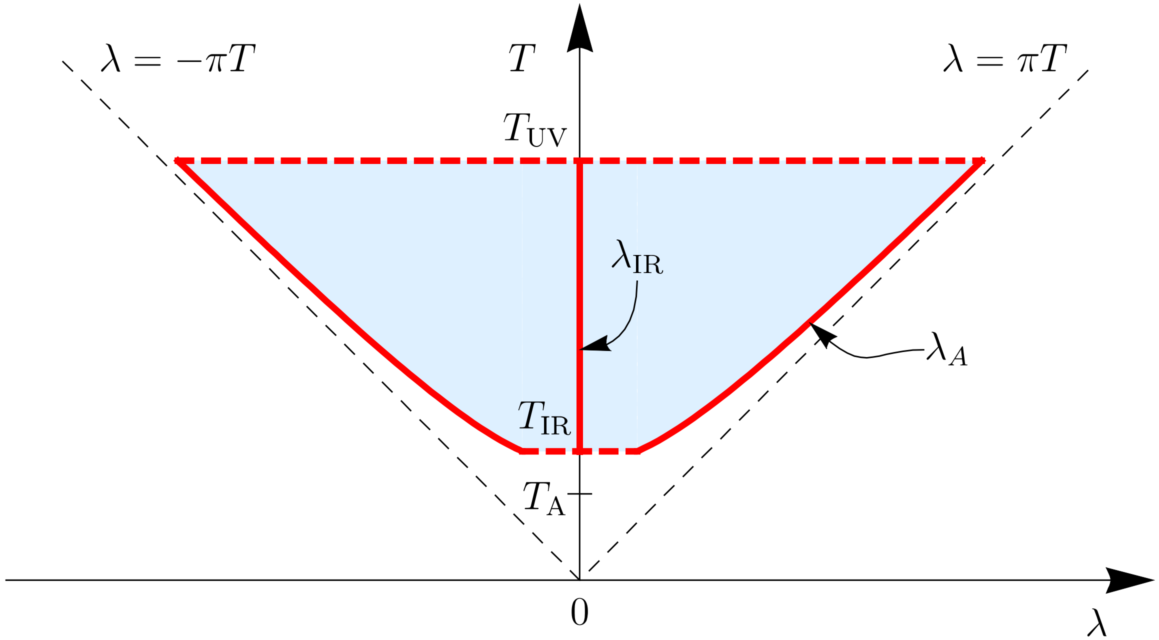
Q: Is there a known non-analytic structure in the Dirac spectrum that could do that?

A: Anderson-like mobility edge $\lambda_A > 0$ proposed/studied in

Garcia-Garcia & Osborn hep-lat/0611019 Kovacs & Pitler 1006.1205 Giordano, Kovacs & Pitler 1312.1179

How did the connection to Anderson-like localization arise? 1906.08047, 2103.05607, 2110.04833

But λ_A not sufficient. Also need a structure facilitating scale invariance in IR! λ_{IR}

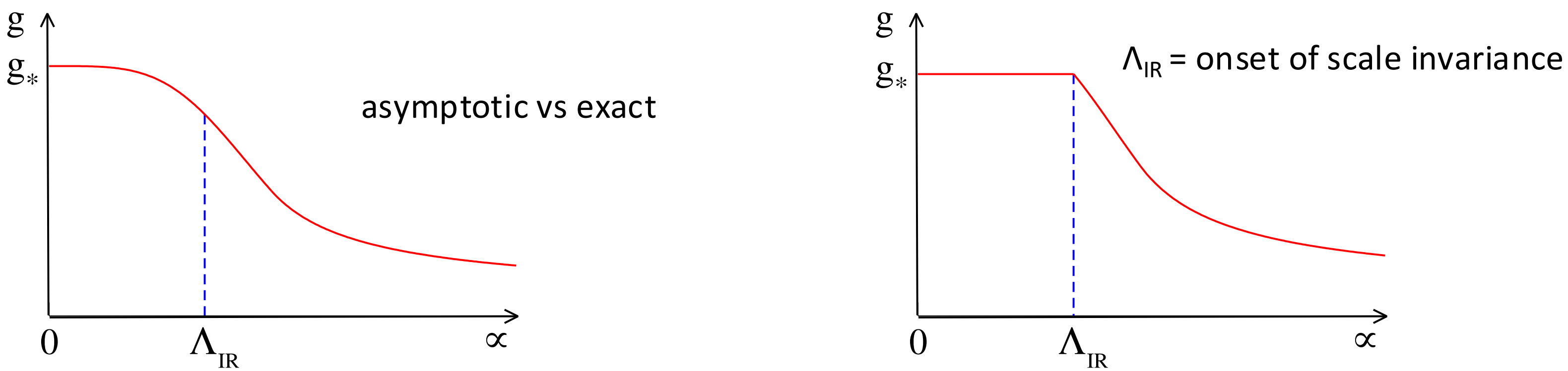


Dirac spectral phase diagram of QCD (pgQCD)

Metal-to-critical scenario 2110.04833 accommodates all needed properties

The new mobility edge λ_{IR}=0 gives long-range physics and the old mobility edge λ_A facilitates decoupling

Model in Kovács 2311.04208 supports decoupling.



Q: Do non-analyticities exist and, if so, how do they arise?

Their existence in λ-dependences would also facilitate IR-BULK decoupling!

Hint: Given the existence of λ_A and its nature, focus on spatial IR dimensions of modes.

WHAT IS IR DIMENSION OF MODES? Concept didn't exist.

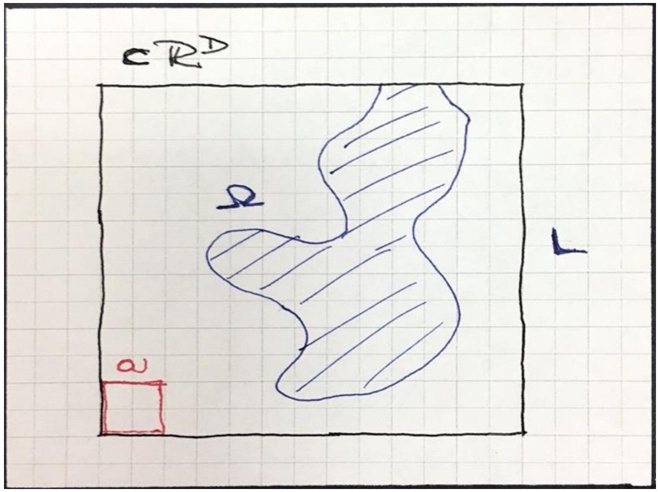
IH & RM 1807.03995 — effective-number theory

IH, PM and RM 2205.11520 — effective-dimension theory

Effective dimensions

A.A. & I.H. 2103.05607

Key to this are eigenmode dimensions: dimensions of space effectively occupied by modes



Two types of dimensions: characterize fine (UV) and global (IR) features

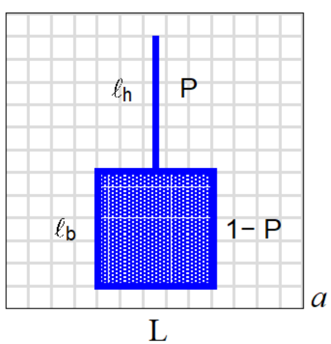
$N \propto (L/a)^D$ N_+ = # cubes covering Ω

UV: $N_+(a,L) \propto a^{-d_{\mathrm{UV}}(L)}$, $a \to 0$

IR: $N_+(a,L) \propto L^{d_{\mathrm{IR}}(a)}$, $L \to \infty$

IR dimensions rarely discussed: obviously that's what we are interested in

Example:



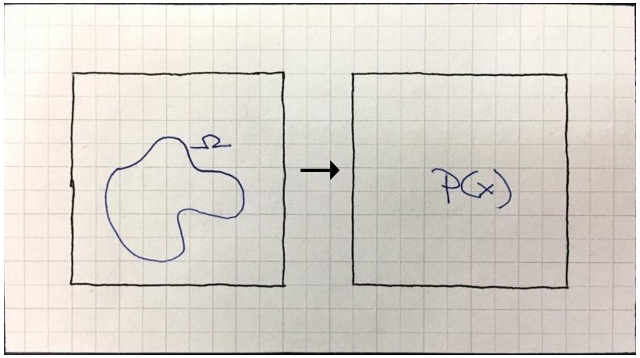
$d_{\mathrm{UV}} = 2$

$\ell_h = \mathrm{const} \longrightarrow d_{\mathrm{IR}} = 0$

$\ell_h \propto L \longrightarrow d_{\mathrm{IR}} = 1$ [shovel reaches everywhere in space]

Effective dimensions

But how to proceed if we only have probabilities?



$$P(x) \quad \Longrightarrow \quad \Omega_{\rm eff}$$

How???

[And we are on the lattice to begin with…]

Since measure has to be assigned to $\Omega_{\rm eff}$ for dimension à la Minkowski/Hausdorff:

1) count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P*
   (must be additive counting in order to obtain measure)

2) select $\Omega_{\rm eff}$ as $\mathcal{N}$ most probable points on the lattice

3) proceed as Minkowski (box-counting)

Consistent realization of the above program leads to unique effective dimension

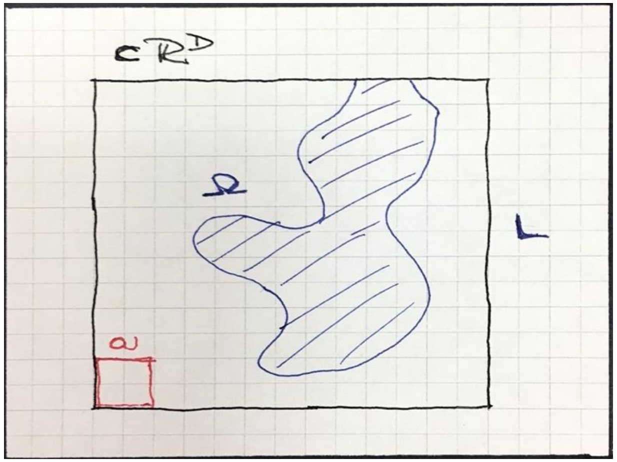
$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$

I.H. and R. M., 1807.03995
I.H. , P. M. and R.M. 2205.11520

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

Defines measure-based dimension for probabilistic sets (effective subsets).

Effective dimensions



characterize both fine (UV) and global (IR) features

$$N \propto (L/a)^D$$

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\mathrm{UV}}(L)} \;,\; a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\mathrm{IR}}(a)} \;,\; L \to \infty$$

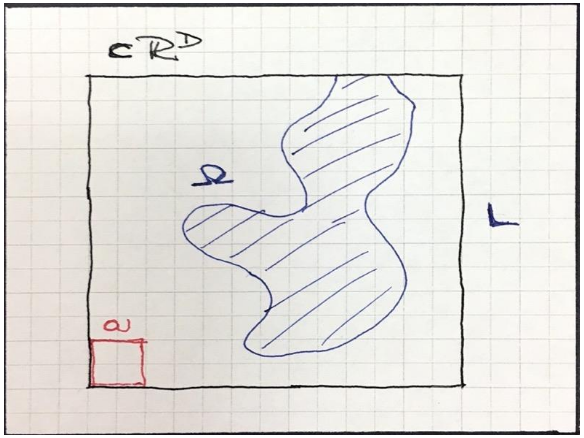
Effective Dimensions of Dirac eigenmodes:

$$\text{UV:} \quad \langle\, \mathcal{N}_\star \,\rangle_{a,L,\lambda} \propto a^{-d_{\mathrm{UV}}(L,\lambda)} \quad , \quad a \to 0$$

$$\text{IR:} \quad \langle\, \mathcal{N}_\star \,\rangle_{a,L,\lambda} \propto L^{d_{\mathrm{IR}}(a,\lambda)} \quad , \quad L \to \infty$$

Finite T: $N = \left(\frac{L}{a}\right)^3 \frac{1}{Ta}$ $\qquad P = (p_1, p_2, \ldots, p_N) \;,\; p_i = \psi_\lambda^+ \psi_\lambda(x_i)$

Effective Dimensions



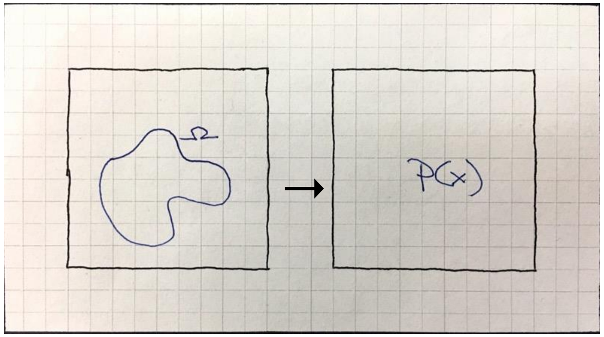
characterize fine [UV] and global [IR] features of fixed sets

all points/elements of regularized space: $N \propto (L/a)^D$

points/elements covering Ω: N_+

$$\text{UV:} \quad N_+(a,L) \propto a^{-d_{\rm UV}(L)} \;\;, \;\; a \to 0$$

$$\text{IR:} \quad N_+(a,L) \propto L^{d_{\rm IR}(a)} \;\;, \;\; L \to \infty$$



$$P(x) \quad \Longrightarrow \quad \Omega_{\rm eff}$$

But how to proceed when instead of fixed Ω we have P(x)?

1) Count how many points $\mathcal{N} = \mathcal{N}[P] = \mathcal{N}(p_1, p_2, \ldots, p_N)$ are effectively selected by *P*.

2) Select $\Omega_{\rm eff}$ as $\mathcal{N}$ most probable points on the lattice

3) Proceed as Minkowski/box-counting with N_+

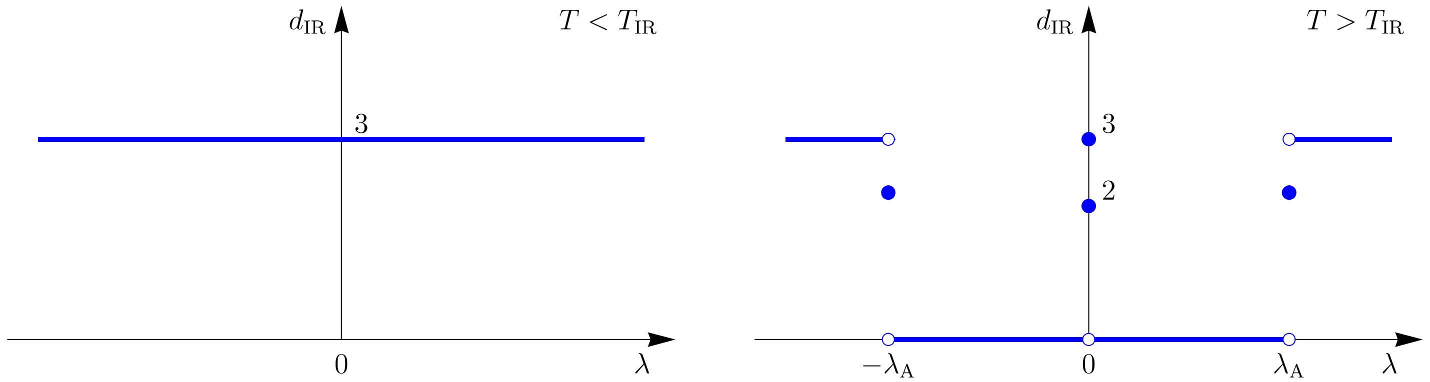
Consistent realization of this program leads to unique effective dimensions   IH, PM and RM 2205.11520

$$\mathcal{N}_\star[P] = \sum_{i=1}^{N} \mathfrak{n}_\star(Np_i) \quad , \quad \mathfrak{n}_\star(c) = \min\{c, 1\}$$

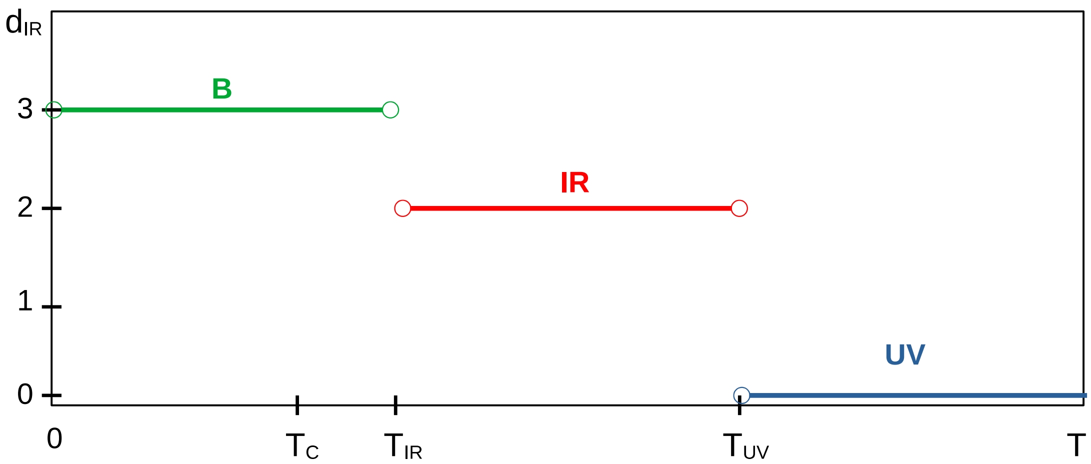
IH & RM 1807.03995

$$\text{Box:} \quad N \longrightarrow N_+ \qquad \text{Effective:} \quad N \longrightarrow \mathcal{N}_\star[P]$$

Pattern of d_{IR} non-analyticity in IR phase



Pattern of spectral non-analyticity 2103.05607 & 2310.03621



IR phase pattern of temp non-analyticity both in pgQCD and QCD.

Effective sizes

2110.04833

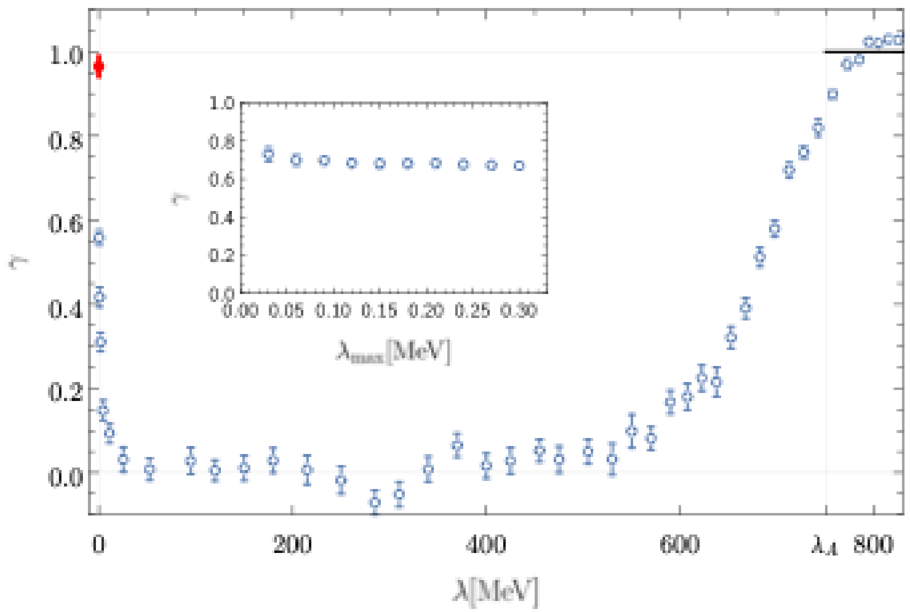
$$\ell[\psi] = \ell[P] = \sum_{i=1}^{N} p_i \, |x_m - x_i| \quad , \quad p_m \geq p_i \ , \ \forall \, i$$

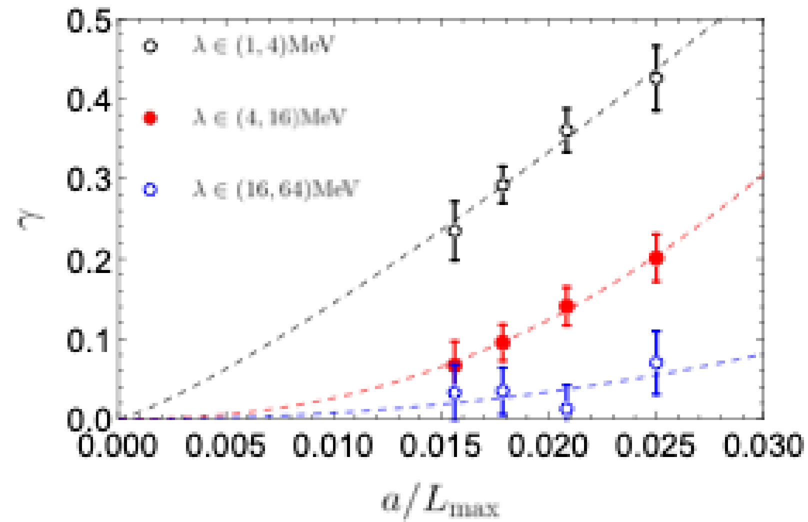
Mean distance from the maximum: simplest, robust…

$$\langle \, \ell[\psi] \, \rangle_{a,L,\lambda} \propto \ L^{\gamma(a,\lambda)} \quad \text{for} \quad L \to \infty$$

Effective sizes...

Similar to d_{IR}





2110.04833

Point: To claim localization both d_{IR} and γ have to be zero...

Wish list

(1) continuum limit of spectral density (e.g.pgQCD 1.1 T_c L=5 fm)

(2) Details of cutoff-dependences (chiral random models) Evangelou etc.
Actually fascinating… How do dimensions emerge from noise?

(3) The issues related to scale invariance are largely resolved conceptually
2509.03509

But decoupling needs a solid numerical demonstration. Formalism ready…

(4) UV dimensions - is the d_{UV} structure also non-trivial?

(5) Hopefully in Pisa we also touch cosmology…

Standard Model of Particle Physics + GR

(1) Do we need unknown/untested physics to understand the cosmological "later"?

Both transitions believed to be analytic crossovers:

(2) Is that all there is or are we missing something?

BOTH QUESTIONS NEED CONVINCING <u>NEW ARGUMENTS</u> AND <u>NEW FACTS</u>!

Focus on QCD transition here…

BACKUPS

A. IR Phase of SU(3) Gauge Theories...

I. IR PHASE OF THERMAL QCD

1906.08047, 2404.12298, 2305.09459

above/different from χ-crossover T_c

$T_c < T_{IR} < T < T_{UV}$

≈155 MeV 200-230 MeV perturb

II. WHY IR?

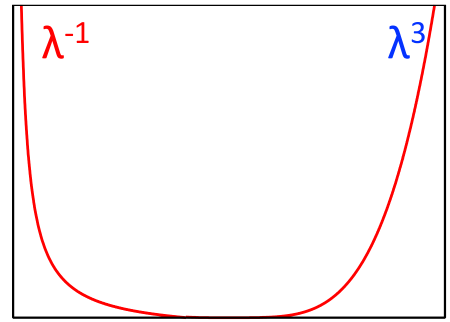
Power-law accumulation of DOFs in the IR AA&IH 1906.08047

Thermal QCD in IR phase:

-highly unusual scales $\Lambda < 1$ MeV

-partial deconfinement 1502.07732

ρ



λ

$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta = \delta(a) \rightarrow 0$?

$a \rightarrow 0$

III. WHY PHASE?

At T_{IR}:

1906.08047

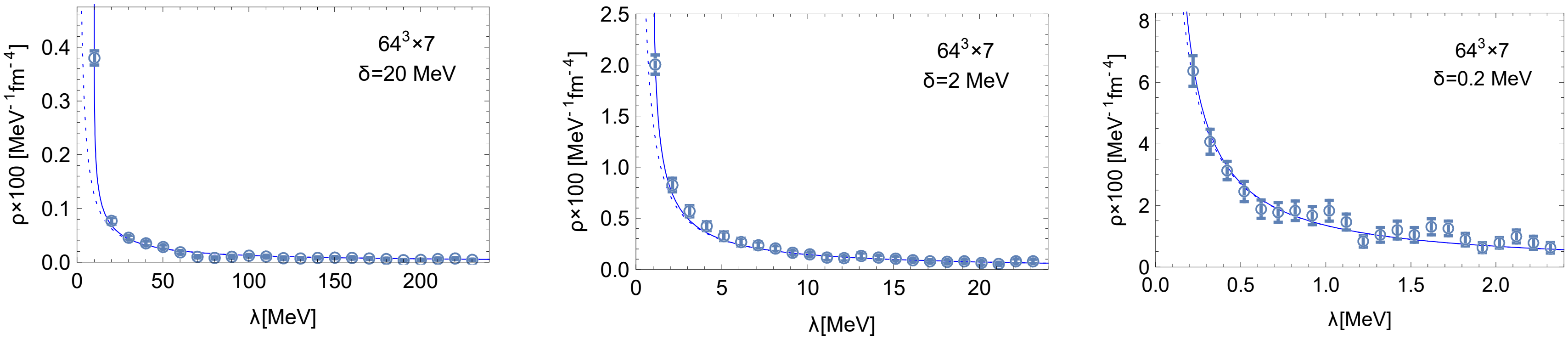
(i) IR BECOMES AN AUTONOMOUS SUBSYSTEM
[IR-BULK decoupling, from1-component to 2-component system]

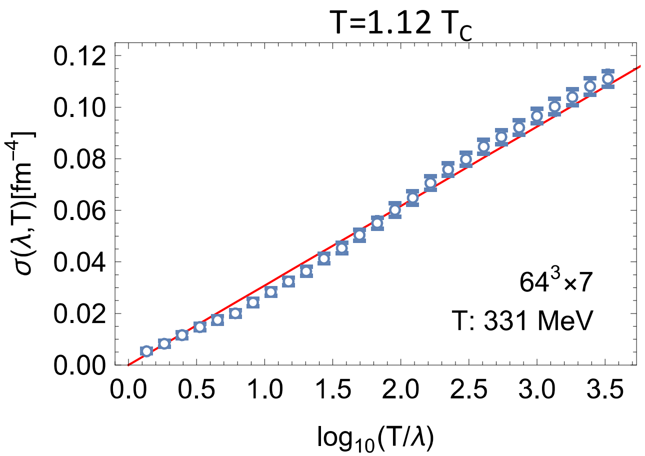
(ii) GLUE OF IR COMPONENT BECOMES SCALE INVARIANT

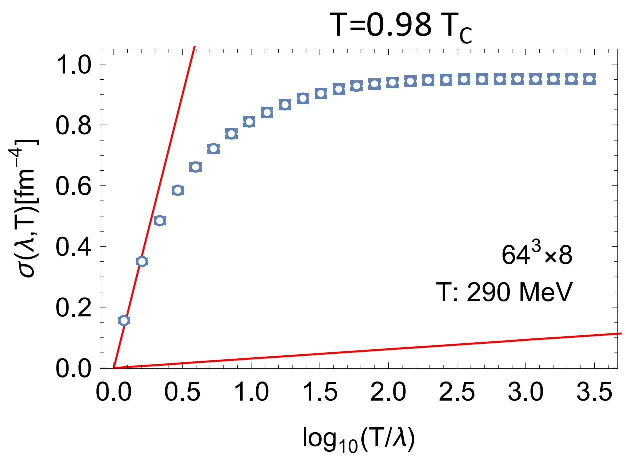
(iii) NON-ANALYTICITIES APPEAR

(iv) INFINITE GLUE SCREENING LENGTHS APPEAR

Fits to $\rho(\lambda) \propto 1/\lambda$ [N_f=0, T=1.12 T_c] AA & IH 1906.08047







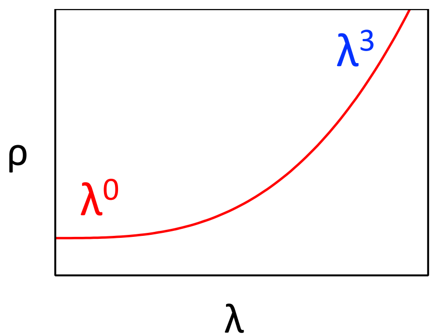
Data: (1) IR SCALE-INVARIANT DENSITY ($\lambda < T$) OVER 3 ORDERS OF MAGNITUDE IN SCALE

(2) NEGATIVE POWER-LAW ACCUMULATION OF DIRAC MODES IN IR: $\rho(\lambda) \propto \lambda^p$ $p \gtrsim -1$

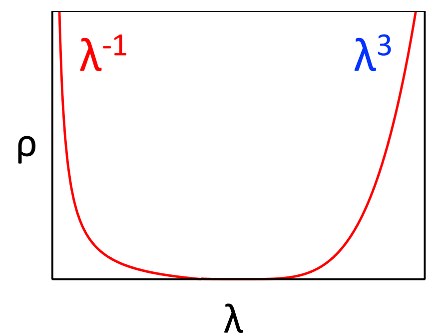
Proposal: THIS REFLECTS IR SCALE-INVARIANT GLUE: <u>IR PHASE</u> [$p<0$] 1906.08047

T=0 classically scale invariant theory — quantum fluctuations ⟹ scale anomaly — scales generated world of hadrons

scale-broken T=0 theory — thermal fluctuations ⟹ increasing T — scale-invariant but only for $\Lambda < \Lambda_{IR} < T$



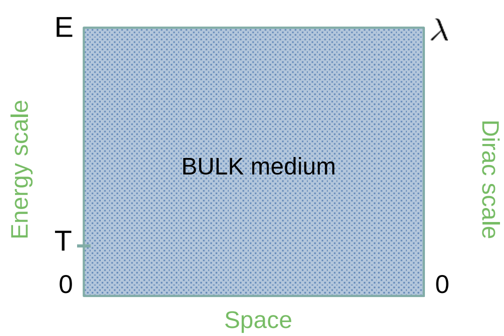
thermal agitation ⟹



$\lambda^{-1} \rightarrow \lambda^{-1+\delta}$

$\delta=\delta(a) \rightarrow 0$?

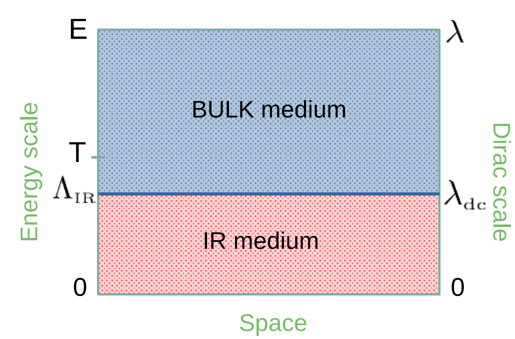
$a \rightarrow 0$



thermal agitation ⟹

IR-BULK SEPARATION

AA & IH 1906.08047



NON-INVARIANT

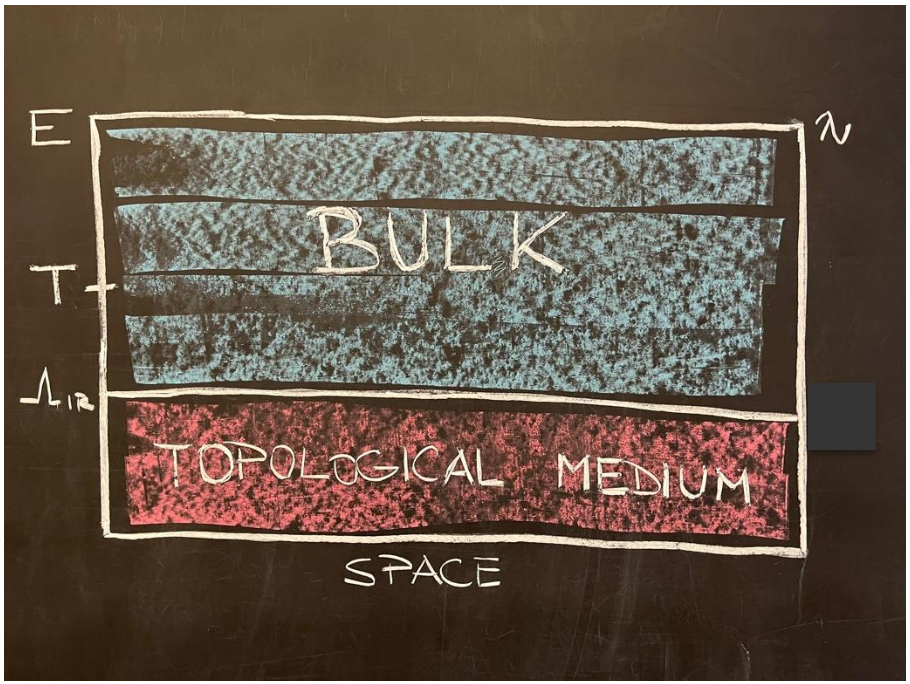
INVARIANT

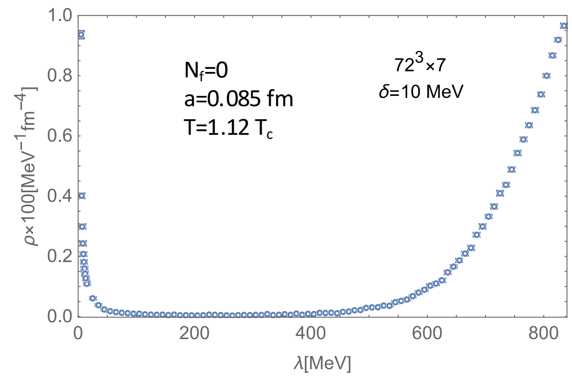
AT $T=T_{IR}$ THERMAL QCD BECOMES 2-COMPONENT SYSTEM: IR MEDIUM AN AUTONOMOUS SUBSYSTEM

Hypothesis: IR phase describes the near-perfect fluid [RHIC, ALICE] state of matter

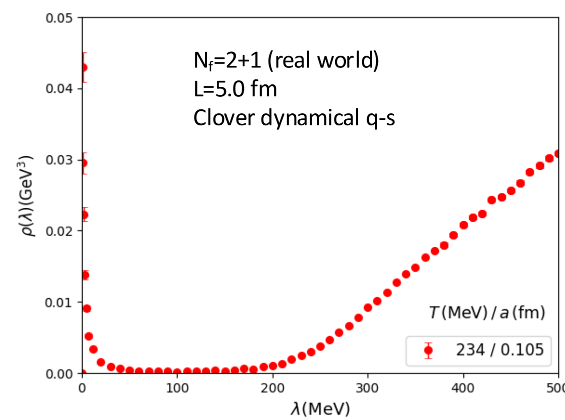
Experimental signatures on ALICE3?

AA & IH 1906.08047





AA & IH unpublished



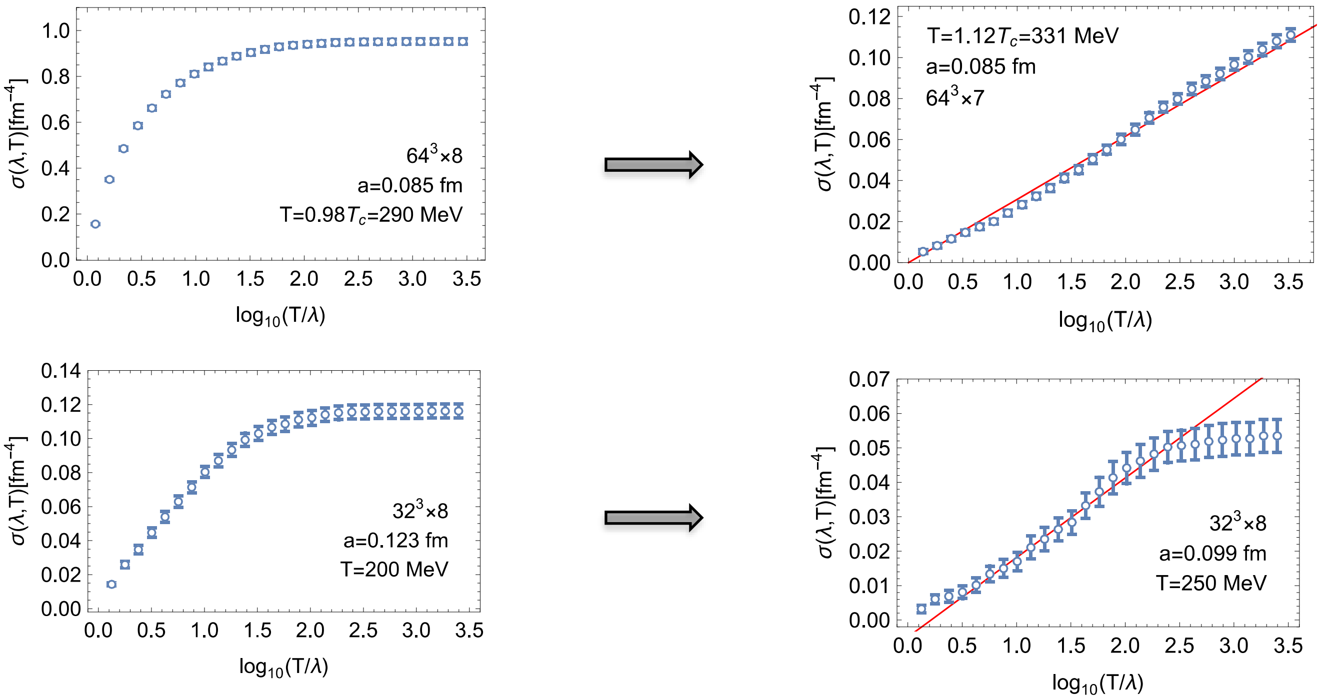
X. Meng et al 2305.09459

- System in IR Phase a 2-component medium: invariant <u>IR</u> and non-invariant <u>bulk</u>
- Tisza/Landau 2-component theory of liquid helium? Maybe in some dual form.

A. IR Phase: Real-World QCD

N_f=0

N_f=2+1



Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks (Wuppertal-Budapest) here.

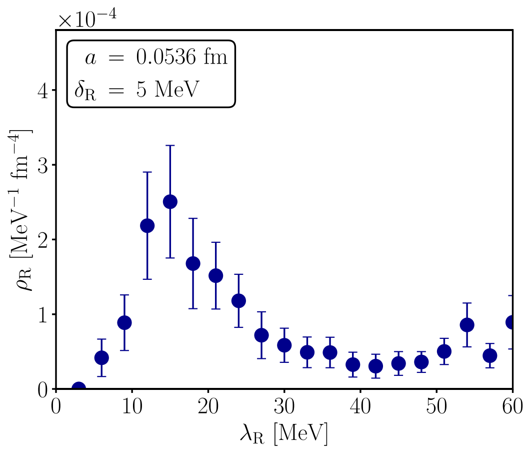
Conjecture: REAL-WORLD QCD HAS IR PHASE WITH p≈-1

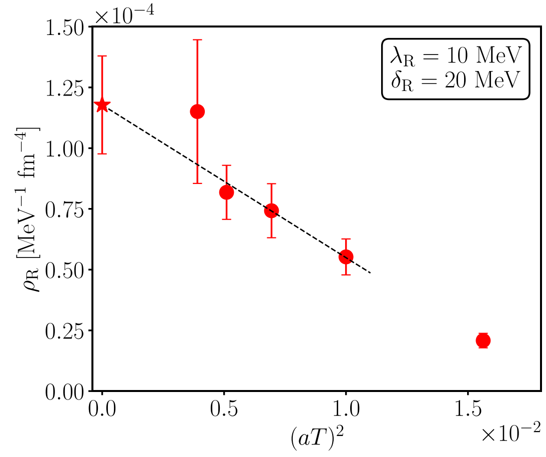
$$200\,\mathrm{MeV} \;<\; T_{\mathrm{IR}} \;<\; 250\,\mathrm{MeV}$$

AA & IH 1906.08047

A. IR Phase: Real-World QCD…

AA, Bonanno, D`Elia, IH 2404.12298





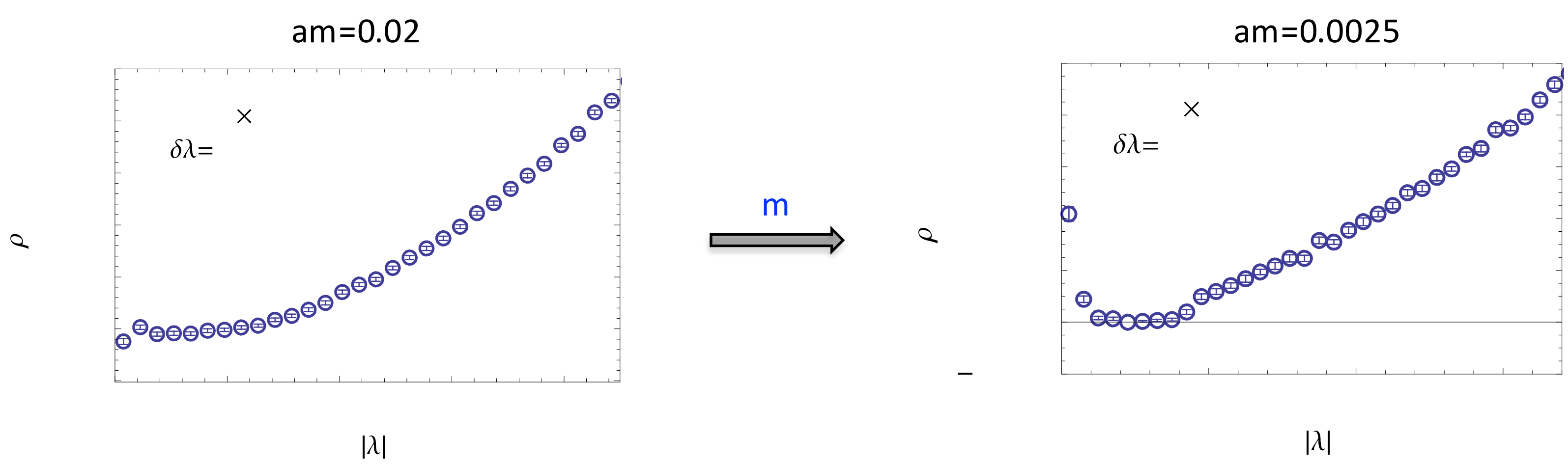
Real-world QCD is N_f=2+1 at physical quark masses of stouted staggered quarks here.

Lattice Dirac operator = stouted staggered [not overlap]

- IR structure exists in Dirac operator describing dynamical quarks
- not a lattice artifact
- IR medium is a quark-glue medium
- green light to study IR phase using overlap: correct & efficient

A. IR Phase: Theories with Many Flavors?

N_f=12, T=0   Configss: A. Hasenfratz et al, 1207.7162   staggered with nHYP   AA & IH 1405.2968 1411.1777

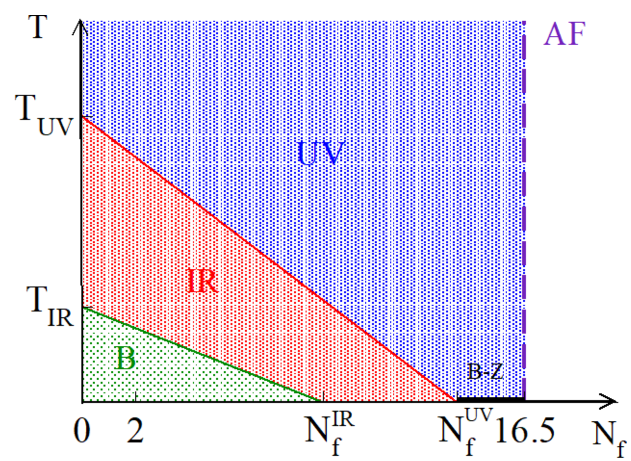


Lowering quark mass at sufficient number of flavors can generate IR phase

Conjecture: AA & IH 1906.08047

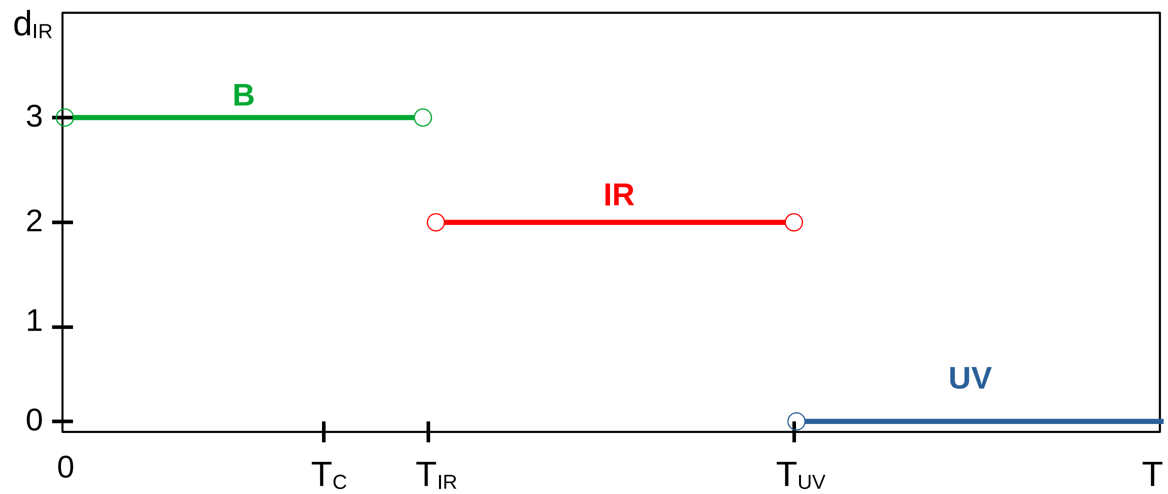
Conformal window has a strongly coupled part with p < 0.

$$N_f^c \equiv N_f^{\rm IR} < N_f < N_f^{\rm UV} \leq 16.5$$



D. Anderson Localization in IR Phase…

Clean representation of non-analyticity in IR dimension: represents d_{IR} of near-zero modes and d_{IR} of IR glue field strength



In which aspects of IR phase are Anderson-like localization features relevant?

(i) IR BECOMES AN AUTONOMOUS SUBSYSTEM YES

[IR-BULK decoupling, from1-component to 2-component system]

(ii) GLUE OF IR COMPONENT BECOMES SCALE INVARIANT YES

(iii) NON-ANALYTICITIES APPEAR YES

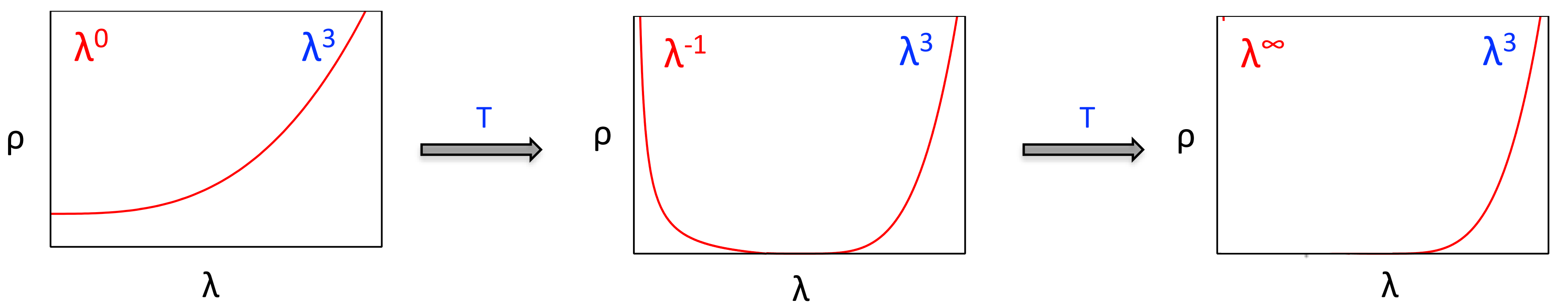
(iv) INFINITE GLUE SCREENING LENGTHS APPEAR YES

WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION

N_F=0 easier to communicate the point [same in N_f=2+1 just more awkward.]

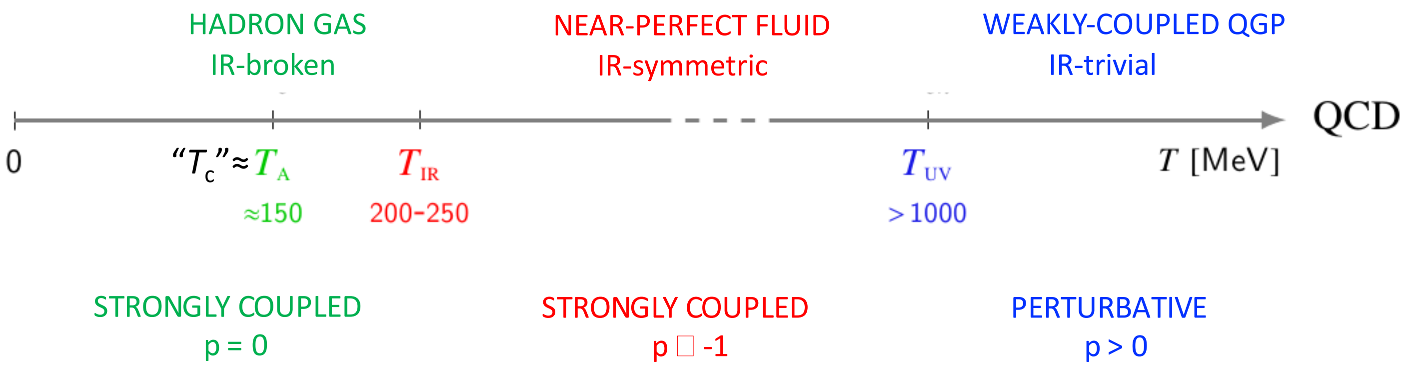
T=0 classically scale invariant theory → (quantum fluctuations / scale anomaly) → scales generated world of hadrons etc

scale-broken T=0 theory → (thermal fluctuations / increasing T) → scale-invariant but only for $\Lambda \leq \Lambda_{\rm IR} < T$



WHAT WE HAVE HERE IS THE LACK OF COMMUNICATION…

PHASE STRUCTURE OF THERMAL QCD IN TERMS OF GLUE IR SCALE INVARIANCE [AA & IH 1906.08047]

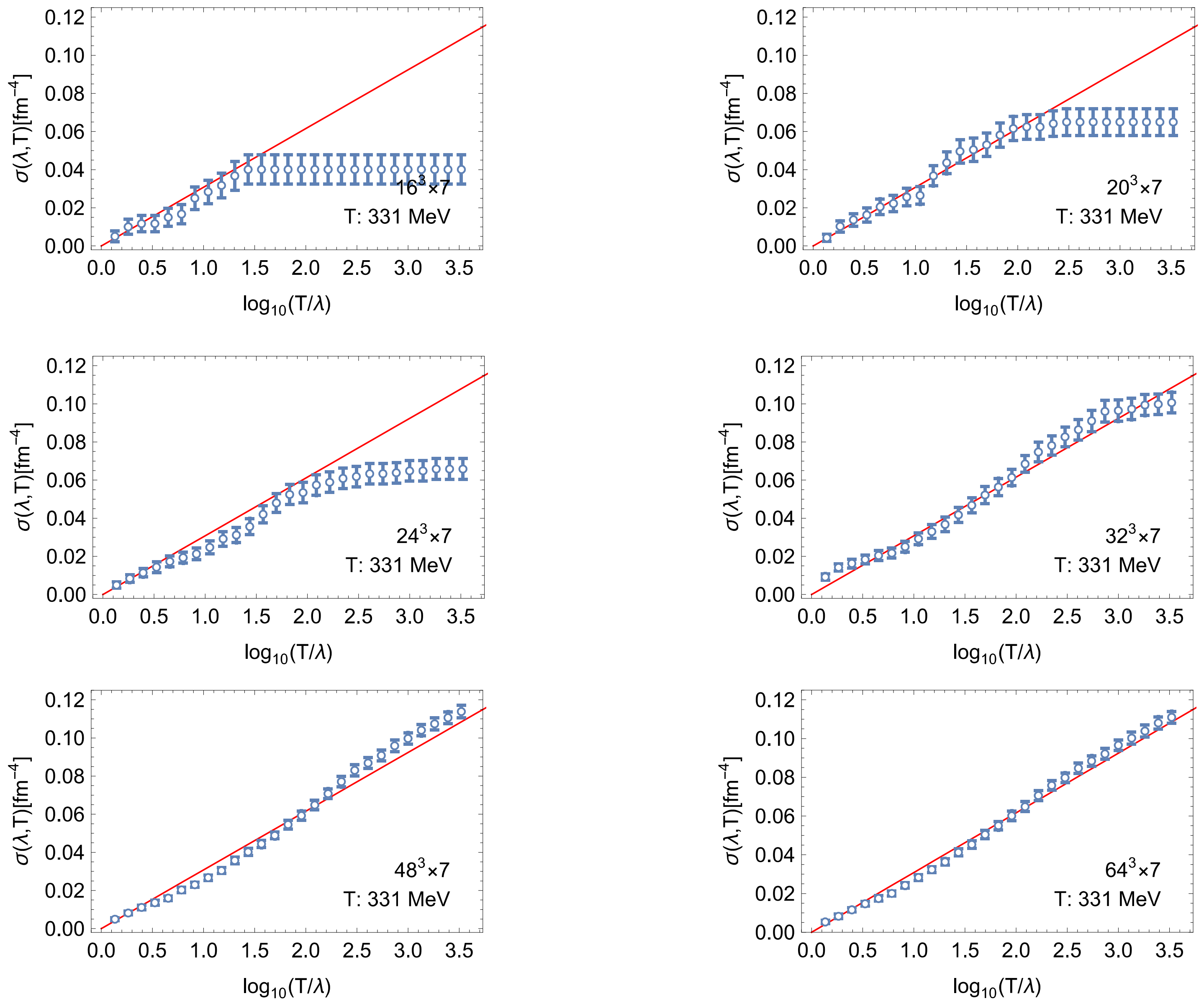


$$\text{phase} = \begin{cases} \text{B} & \text{if} \quad p = 0 \\ \text{IR} & \text{if} \quad p < 0 \\ \text{UV} & \text{if} \quad p > 0 \end{cases} \qquad \text{with} \qquad \rho(\lambda) \propto \lambda^p \quad \text{for} \quad \lambda \to 0$$

Original talk: https://indico.cern.ch/event/764552/contributions/3420459/attachments/1865996/3068382/WuHan_jun_2019_infra.pdf

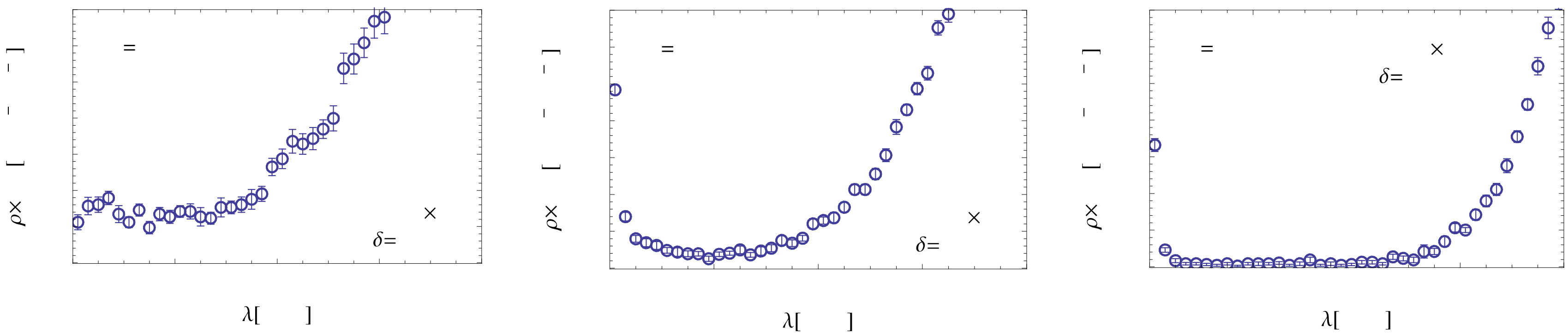
Useful talk: https://drive.google.com/file/d/1vZ0AY0WsZAfF9iV7-Br-E_2NiwaZzRGp/view

Check this further & better… $\sigma(\lambda_1, \lambda_2) \equiv \int_{\lambda_1}^{\lambda_2} d\lambda\, \rho(\lambda)$ $\rho(\lambda) = c/\lambda \longrightarrow \sigma(\lambda, T) = c \ln(T/\lambda)$



- Peak in IR overlap spectrum upon crossing T_c (pure glue) [Edwards, Heller, Narayanan, Kiskis, 1999]

- Our version of it [AA & IH, 1502.07732]



  - knee-jerk reaction was: quenched chiral condensate may diverge in high-temperature pure glue
  - knee-jerk reaction should be: what on earth is <u>glue</u> doing to produce this? [1502.07732]
  - didn't know but went on with it, e.g., around chiral crossover we got this: