

Effects of $U_A(1)$ and disorder in the Dirac spectrum of finite temperature QCD

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H. Pandey, R.S. and S. Sharma

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- Most systems studied so-far are typically at zero or finite temperature with no thermodynamic phase transitions.

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- However the **interplay between restoration of different subgroups of chiral symmetry breaking and localization** is not very well understood.

Effective restoration of chiral symmetries in QCD

- The non-singlet subgroup of the chiral symmetry i.e. $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$ is effectively restored beyond temperature $T_{pc} = 156.5 \pm 1.5$ MeV [HotQCD collaboration, 2018]

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- Singlet $U_A(1)$ subgroup is anomalous [Adler, Bell, Jackiw, 1969, Fujikawa, 1980] hence there is **no order parameter**.
- We can only discuss its **effective restoration** at $T_{U(1)} \sim 1.5 T_{pc} \rightarrow$ where $\chi_{\text{top}} \sim T^{-8}$ consistent with the dilute instanton gas model prediction
[P. Petreczky, H-P Schadler, S. Sharma, PLB 2016, S. Borsanyi et. al., Nature 539, 2016]
or though the **degeneracy of meson correlators** [G. Aarts et. al. , 2604.1191].

Properties of the Dirac eigenstates above T_{pc}

- In quenched QCD, Anderson-like localization of infrared Dirac eigenvalues has been discussed.

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- **Many different intertwined phenomena are happening in the regime $T_{pc} < T < 1.5 T_{pc}$.**

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- Since $U_A(1)$ subgroup of chiral symmetry is **anomalous** we propose to identify the effective restoration of $U_A(1)$ through the **structural properties** of infrared Dirac eigenstates.

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- We will also demonstrate that only well above the temperature regime where $U_A(1)$ is effectively restored, one can **observe localization** in the infrared eigenstates.
- Can we learn about thermalization from the properties of Dirac eigenspectrum?

Numerical set-up

- The thermal gauge configurations for $T = 164, 177, 195, 270, 314, 339$ MeV were generated using Möbius domain-wall fermion discretization for 2+1 flavors of dynamical physical quarks and Iwasaki gauge action.
- We calculate first 100-200 eigenvalues of the massless overlap Dirac operator on these thermal gauge configurations
- The chiral crossover transition happens at $T_{pc} \simeq 158$ MeV.
[M. E. Jaensch, R.S. et. al., Phys. RevD. 111, 034507 (2024)].
- The spatial lattice size in physical units varies between 4-2.5 fm for $T = 164-339$ MeV.
- We use the Wilson flow renormalization scheme following $a \ll \sqrt{8t} \ll 1/T$ to remove the UV divergences while calculating expectation values of the Polyakov loop.

Observables

- We calculate the normalized level ratios,

$$\tilde{r}_n = \min\left(r_n, \frac{1}{r_n}\right), \text{ where } r_n = \frac{s_{n+1}}{s_n}$$

in terms of the spacings $s_n = \lambda_{n+1} - \lambda_n$ between two consecutive Dirac eigenvalues λ_n .

- We study $\langle \tilde{r} \rangle$ in small bins in λ . These observables are free from systematic errors related to unfolding.
- It is well known that universal level-spacing fluctuations of Dirac eigenvalues at $T < T_{pc}$ can be described in terms of random matrices belonging to Gaussian unitary ensemble (GUE).

[J. J. M. Verbaarschot, Phys. Rev. Lett. 72, 1994]

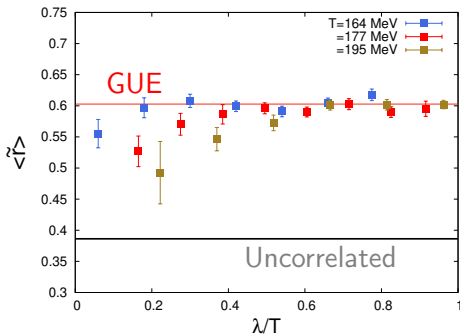
- We will instead study the level ratios at $T > T_{pc}$ and use it to categorize different regions of the Dirac eigenvalue spectrum.

Level ratios of Dirac eigenvalues as for $T \gtrsim T_{pc}$

- In most λ/T bins the values $\langle \tilde{r} \rangle$ are consistent with the GUE prediction $\rightarrow$ **bulk eigenmodes**.

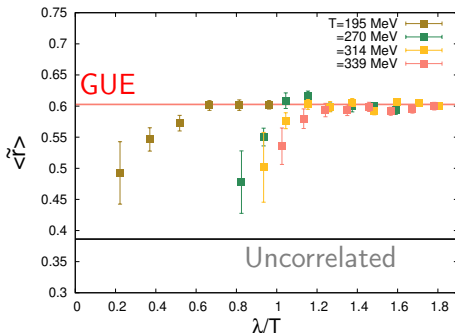
[Plots from R.S., H. Pandey, S. Sharma, 2602.24227].

- Few eigenmodes appearing in the deep infrared part of the spectrum have level ratio that is intermediate between GUE and uncorrelated ensembles $\rightarrow$ **intermediate eigenmodes**.



Level ratios of Dirac eigenvalues for $T \gtrsim T_{U(1)}$

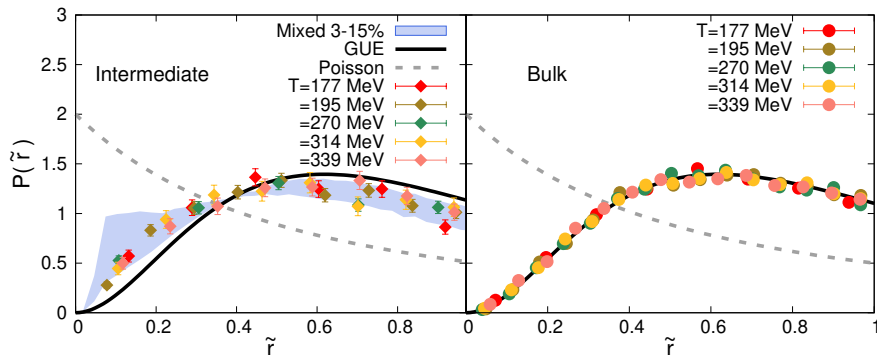
- A gap in the infrared appears at $T_{U_A(1)} \sim 1.5 T_c$.
- Intermediate eigenvalues still persist beyond the gap but become more rarer with increasing T .



A random matrix model for the intermediate eigenmodes

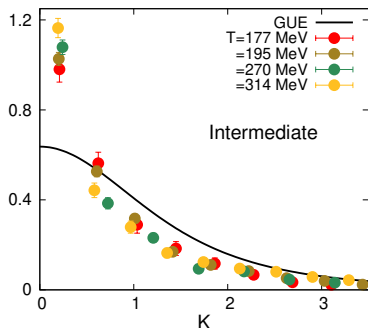
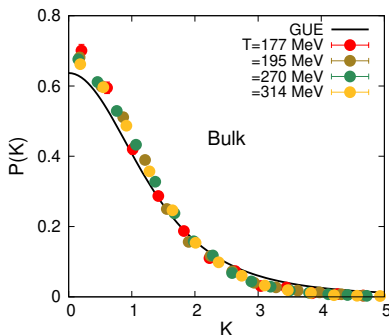
- For the **intermediate eigenmodes** we proposed a matrix model where a large matrix belonging to GUE has a **3-15%** admixture of eigenvalues with Poisson statistics.

[R.S., H. Pandey, S. Sharma, 2602.24227]

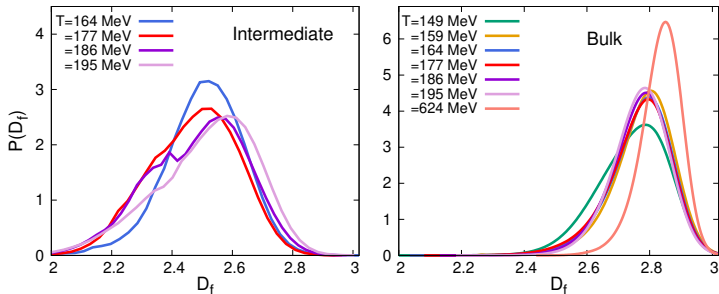


Structural features of the Dirac eigenfunctions

- We next monitor how Dirac eigenstates change due to a twist applied on one of the spatial boundaries $\psi(x + L) = e^{i\eta}\psi(x)$.
- Curvature $K_i = \lim_{\eta \rightarrow 0} \left| \frac{\partial^2 \lambda_i}{\partial \eta^2} \right|$. for the bulk and intermediate eigenmodes are very different.

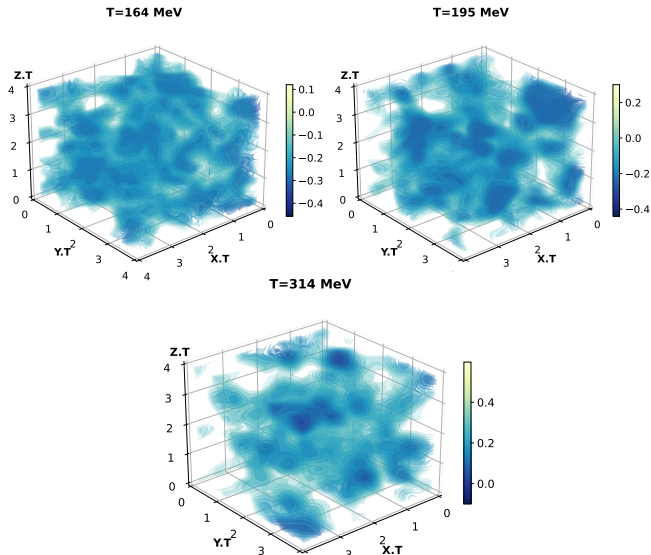


Intermediate eigenmodes at $T \gtrsim T_{pc}$ and criticality



- The fractal dimensions of intermediate modes $D_f \sim 2.5$. Using the fact that $D_f = 3 - \frac{\beta}{\nu}$ [K. Jansen and C. B. Lang, Phys. Rev. Lett. 66, 1991], we can interpret the value of D_f using β and ν , the critical exponents of $O(4)$ model.
- Indicates that $U_A(1)$ is not effectively restored!

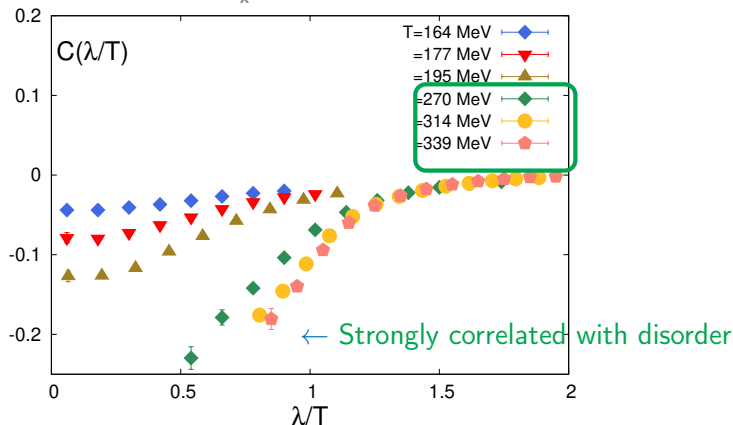
Disorder in the gauge fields quantified in terms of Polyakov loop fluctuations



Change in localization properties of intermediate IR eigenmodes due to disorder

- The correlation of Dirac eigenmodes with the local fluctuations in the renormalized Polyakov loop values can be quantified in terms of

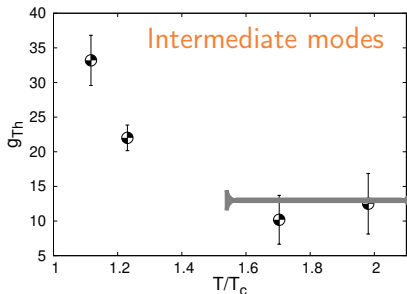
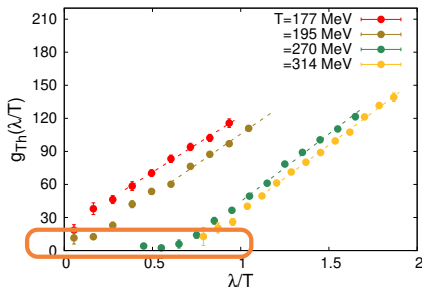
$$C_n = \sum_x |\Psi_n(\mathbf{x})|^2 [\text{Re}.P(\mathbf{x}) - \langle \text{Re}.P(\mathbf{x}) \rangle].$$



Thouless conductance as an effective order parameter for $U_A(1)$?

- We calculate the **Thouless conductance** which measures structural rigidity of eigenvectors

$$g_{\text{Th}} = \frac{1}{\langle s_i \rangle} \langle K_i \rangle_{\eta \rightarrow 0} \quad \text{where} \quad K_i = \left| \frac{\partial^2 \lambda_i}{\partial \eta^2} \right|.$$

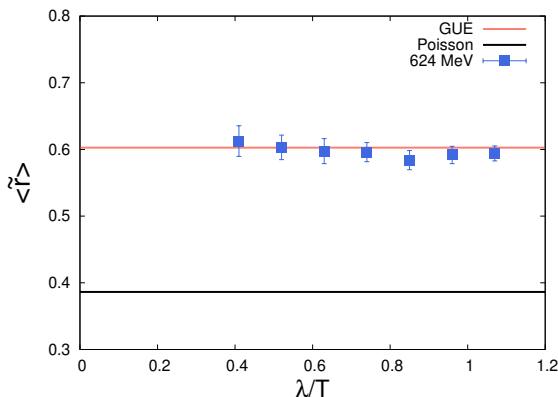


What more can the Dirac spectrum tell us: thermalization

- Thermalization in an isolated classical system is closely related to the ideas of chaos and ergodicity [See for e.g Review by J. M. Deutsch, 2018].
- Chaos defined in terms of exponential separation between two initial close trajectories in phase space is no longer valid for quantum systems.
- The first major attempt to understand quantum analogs of classical chaos was by Bohigas, Giannoni, and Schmit (BGS, 1984).
- They conjectured that **spectral fluctuations of quantum systems whose classical dynamics are chaotic** follow the predictions from one of the three classes of random matrix theory (RMT).

A demonstration of the BGS conjecture

- At $T \sim 600$ MeV all infrared Dirac eigenvalues in quenched QCD below the magnetic scale $g^2 T/\pi$ has level spacing ratios consistent with GUE.



A demonstration of the BGS conjecture in QCD

- We have also constructed a non-thermal state consisting of $SU(3)$ gluons whose infrared momentum modes have occupation numbers $\gg 1$. The state is thus classical and can be evolved using the Hamilton's equation of motion.

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- We next calculate the gauge-invariant distance measure

$$D(U_I, U'_I, t) = \frac{1}{N_P} \sum_P \frac{1}{N_c} |\text{tr} U_P - \text{tr} U'_P|, \quad [\text{B. Müller \& A. Traynov, 1992}]$$

defined in terms of plaquettes U_P , starting from two initial states that have infinitesimally close initial occupation numbers.

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defined in terms of plaquettes U_P , starting from two initial states that have infinitesimally close initial occupation numbers.

- We could extract a positive Lyapunov exponent for different choices of initial occupation numbers $\rightarrow$ characteristic of a chaotic system. This is a non-trivial realization of the BGS conjecture for low-momentum gluons.

[H. Pandey, R. Shanker, S. Sharma, Nucl. Phys. B 1025, 2026].

Summary and Outlook

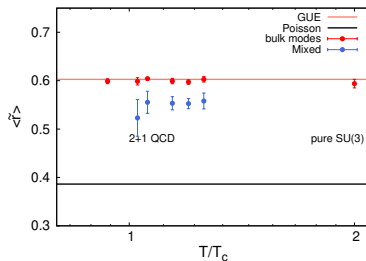
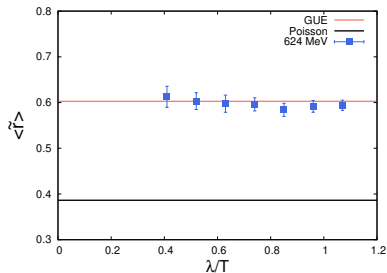
- Appearance of IR Dirac eigenmodes with **intermediate level statistics** has a different physical origin depending on whether or not the $U_A(1)$ is effectively restored.
- These intermediate eigenstates at $T \gtrsim T_{pc}$ carry information about the criticality of the phase transition $\rightarrow$ likely to be $O(4)$.
- We have observed a drop of Thouless conductance and saturating at $T > T_{U_A(1)} \rightarrow$ acts as an **effective order parameter for $U_A(1)$ breaking**.
- At $T > T_{U_A(1)}$ these **intermediate** IR eigenmodes are correlated with random disorder due to wells in the local Polyakov loop values.
- We also discussed a non-trivial realization of the BGS conjecture in quenched QCD at $T \sim 600$ MeV. Has implications for understanding thermalization of strongly interacting low-momentum gluons.

Thank you!

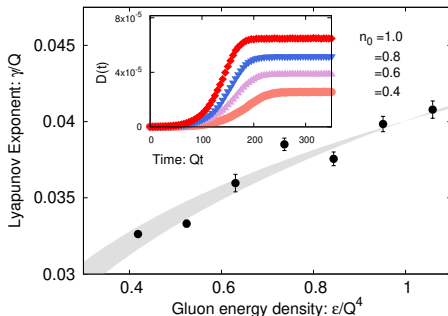
Backup: Matrix model

- $\lambda_0, \lambda_1, \dots, \lambda_{n-1}, \lambda_n, \lambda_{n'}, \lambda_{n+1}, \lambda_{n+2}, \dots, \lambda_{N-1}$
- Here λ_i , where $i = 0, \dots, N-1$ are eigenvalues of our original GUE-block of size N . The three new level ratios that arise as a result are $r_{n1} = \frac{\lambda_{n'} - \lambda_n}{\lambda_n - \lambda_{n-1}}$, $r_{n2} = \frac{\lambda_{n+1} - \lambda_{n'}}{\lambda_{n'} - \lambda_n}$ and $r_{n3} = \frac{\lambda_{n+2} - \lambda_{n+1}}{\lambda_{n+1} - \lambda_{n'}}$, which are normalized to give us $\tilde{r}_{n_i} = \min\left(r_{n_i}, \frac{1}{r_{n_i}}\right)$, $i = 1, 2, 3$.
- The new eigenvalues are accepted only if all three locally induced $\tilde{r}$ are compatible with the target distribution.

Backup: Level ratios of intermediate and bulk eigenmodes versus T

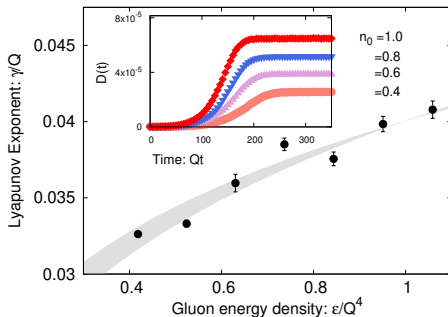


Chaotic nature of a classical non-thermal state of QCD



- The gauge links $U_{i,x}$ and the electric fields $E_{a,x}^i$ are evolved in time according to classical Hamiltonian equations of motion.
- Rapid memory loss $\rightarrow$ enters a self-similar scaling regime.
- Gluon distribution function exhibits a scaling relation of the form,
$$g^2 f_g(|p|, t) = (Qt)^{-\frac{4}{7}} f_s \left[(Qt)^{-\frac{1}{7}} \frac{|p|}{Q} \right].$$

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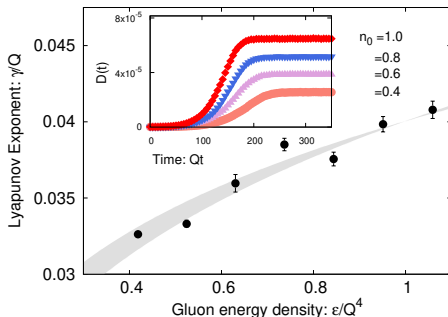


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between two close gauge-trajectories which at initial time $t = 0$ are characterized by n_0 and $n_0 + \Delta n_0$, $\Delta n_0 = 0.001$.

Chaotic nature of a classical non-thermal state of QCD



- System exhibits chaotic behavior resulting in $D(t)$ increasing exponentially with time.
- The positive Lyapunov exponent γ is characteristic of a chaotic system.